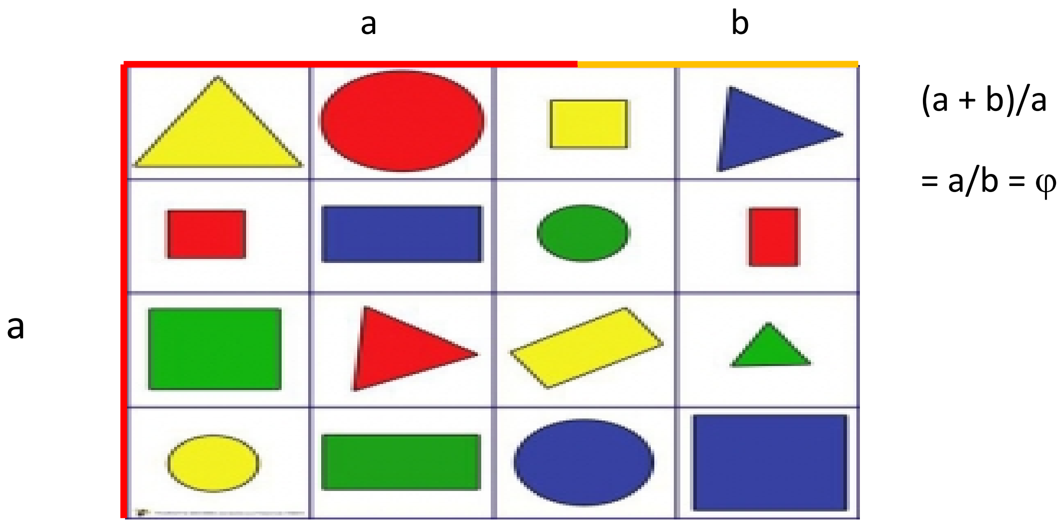
Researching learning and teaching

Is the Van Hiele Model useful in determining how children learn Geometry?



By Sam Curran

Acknowledgements

I am indebted to many people in helping me to compose this dissertation, of which there are a few significant individuals I want to identify for special commendation here.

Firstly, to my lecturers Ian Wood and Fiona Lawton who have provided me with invaluable support in both the formulation and production of my research study. Both have furthered me professionally; I also collaborated with Ian over an article which has since been published in the *Association of Teaching Mathematics* (ATM) Journal.

I would especially like to thank my former A Level Mathematics Teacher Elizabeth Best who has been an inspirational mentor, who further sparked my interest in Mathematics and made me decide to go into a career in education. She also gave me the idea to create a book on Mathematical exercises to use within the classroom which I am hoping to publish.

Thirdly, I would also like to everyone on the Secondary Mathematics QTS degree course who have always believed and supported in me; in particular Cameron Swindells, Jonathan Carver, Josh Philppotts and Becky Hamilton.

Finally and most significantly, I would like to thank my immediate family. My Mother for fostering my interest in education and always believing in and encouraging me to keep going through adversity and my Father for maturing me. Also, I would like to thank my Sister for helping raise me as a child and being the best possible sibling. They have been great friends and have always been there for me.

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Is the Van Hiele Model useful in Determining how Children learn Geometry?

Research paper

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Abstract

The aim of this study is to investigate how children learn Geometry (at all levels of compulsory education) in Mathematics. This study was chosen because of my difficulties in the area and the possible under-representation of Geometry in the Mathematics Curriculum. Five tasks were given to two students for each Key Stage 1-5 inclusive. These were then analysed using the Van Hiele model of Geometric reasoning; which was used to make an assessment of children's geometrical ability. The study also draws on theoretical frameworks from eminent researchers like Vygotsky, Piaget and Bruner as well as engaging fully with current educational literature and research. A questionnaire on Geometry was also completed by a variety of primary, secondary and A-level mathematics teachers. It was found that geometrical ability increases with age (although young children can display sophisticated knowledge of shape) and that students mainly drew shapes of a non-prototypical orientation. This has increased my subject knowledge and enhanced my classroom practice and also may have the implication of changing other practitioners' teaching strategies.

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Introduction

The Department for Business, Innovation and Skills (DBIS) (2012) report that one in four adults have Mathematics skills which are deficient to those of primary school age and less than 60% of all pupils in the 2012 GCSE Mathematics cohort achieved a C or above (DfE, 2013 a). This arguably indicates that numeracy skills are not ideal in the current environment; so it may be beneficial to gain a detailed knowledge of how pupils learn so standards and attainment can be increased.

Mathematics could be perceived as ‘pure’- with more emphasis being given to historically eminent topics like Number and Algebra, as opposed to ‘applied’ modules like Geometry. Johnston-Wilder and Mason (2005) suggest that Geometry is given less teaching time in the classroom than other disciplines. There is also tangible evidence to suggest that this is inherent at all levels of mathematical study: Geometry forms none of the syllabus for the ITT Numeracy QTS Skills Tests (DfE, 2013 b) and is only present in 3 of the 9 attainment descriptors (heavily in Trigonometry but only lightly in Vectors and Logarithmic Functions) at A-Level (CIE, 2013). Furthermore, it has less teaching time than the strands of Number, Algebra and Handling Data in the National Curriculum of both Primary and Secondary School Mathematics (DfE 2011 a; 2013 c). This perceived underrepresentation does not appear to be amended in proposed curriculum reforms; Geometry forms less than a quarter of the amalgamated attainment descriptors in the draft of the 2014 Secondary Mathematics curriculum (DfE, 2013 d).

This was validated in my own experiences in learning Mathematics at school. I have few recollections of studying shape topics; many of my lessons were orientated on Number and Algebra. This study is of particular meaning to me as I experienced many difficulties in learning shape at school and developed an emotional and mental block on it which still persists to this day.

Senechal (1990) states that shape is a vital and key component in learning Mathematics and, if properly developed, can aid cross-curricular links to Science and more creative subjects. Cuoco, Goldenberg and Mark (2012) propose that thinking geometrically seems to provide an alternate perspective on life, investigations and problem-solving. Used in conjunction with a solid understanding of the ‘core’ concepts of Mathematics like Number and Algebra, this could provide a young person with real benefits in life.

For all of these reasons, I decided to conduct a research study investigating how children learn shape. By undertaking this study, I hope to increase my subject content knowledge as well as enhance and aid my classroom instruction. It could also possibly influence other practitioners’ teaching strategies and help an innumerable amount of pupils.

Literature Review

This literature review will focus on the eminent and recent literature which examines the usefulness of the Van Hiele Model in assessing children's geometrical ability and theories of how children learn Geometry.

Piaget (1953; 1960; 1967) suggests that a child's initial geometrical discoveries are *topological*; that they can recognise the boundary aspect of space and distinguish between open and closed figures from the age of 3. Piaget (1953) suggests this development seems to be formulated during the latter sub stages (*tertiary, circular reactions, curiously and novelty*) of the formative *sensorimotor stage* when a child interacts with the world around them and begins to explore the properties of new objects. Bruner (1961, p.21) reaffirms this by proposing that children learn by exploring their surroundings and physical environment.

It could be argued that Bruner (1961, p.23) however places more importance on social learning than Piaget. Vygotsky (1962; 78) implies that children learn in a social constructivist model from *More Knowledgeable Others* (MKOs) and their peers in a classroom environment. Chazan and Lehrer (2012) suggest this is particularly evident in an interactive classroom setting. This seems to be an underlying criticism of Piaget's theory of cognitive development: that he fails to recognise the social aspect of learning. Donaldson (1979) goes further in her criticisms of the findings arising from Piaget's experiments by stating that his experiments were not appropriate and that children did not understand what the tasks required to them to do. Hughes (1986) supports this and also states that due to the arrangements of the task, children were limited to *egocentricity* and could not see another viewpoint. However, Glaserfeld (1995) refutes these criticisms and attributes the children's lack of understanding to the conceptual difference of the mistranslated Piaget text. Regardless of the agreement of the various cognition theories, there seems to be some truth that children learn Geometry in a social manner, at least partially. DfE (2012) identify that social learning is particularly prevalent when a child starts formal education and learns from teacher exposition and interactions with their peers. However, DfE (2009) suggest that the role of More Knowledgeable Others (such as teachers) are more important in a child's geometrical development than their fellow pupils, particularly in practical 'hands –on' topics like measures and mensuration.

DfE (2011 c) states that the curriculum content of Geometry in KS1 and 2 is *Euclidean Geometry*, as children begin to understand the patterns and properties of 2-D shapes. Both Piaget (1967) and Bruner (1961) allude to the concept of *prototypical images*, where an image of a shape is constructed in a child's mind and stored for later use, although both describe it in different ways. Piaget (1953) theorised that children have *symbolic schemata* which are mental pictures or images or what they have experienced in lessons. Bruner (1966) described this method of remembering images as *iconic.*

Based on research carried out on students in their own mathematics classes as part of composing their doctoral dissertations in 1957 in Utrecht, Netherlands, husband and wife Pierre and Dina Van Hiele (1985) devised a model of geometric levels that children progress through (See Appendix 1, p.53-59 for a more detailed version of the model):

- Level 1 (Visualisation) - Children have knowledge of basic shapes but no comprehension of their properties. They cannot link or compare shapes.
- Level 2 (Analysis) – Children understand the properties of shapes but do not use them in a comparison of shapes.
- Level 3 (Abstraction) – Children can make links between shapes based on their properties and can understand some very simple proofs, although they may struggle on more formal examples.
- Level 4 (Deduction) – Children have a good knowledge of Geometry and can use and apply some formal proofs. Children are likely to reach this level at the end of secondary school.

- Level 5 (Rigor) – This is a level of geometric understanding is equivalent to that of a Mathematician and is unlikely to be reached in compulsory education. People at this level have a deep knowledge of formal proofs and can work confidently in most areas of Geometry.

Van Hiele (1985) states a child’s initial study of Geometry in KS1 is the *visualisation* level (Level 1 in his model), where children can name 2-D and 3-D shapes and recognise them in the real world but possess no knowledge of their interrelating properties.

However, DfE (2011 a) state that children are taught to make connections between shapes from KS1. This seems to imply that children will have some knowledge of the common properties of Euclidean Geometry; in a Piagetian sense by linking it back to previous schemata and also by using visual prototypes to identify other shapes in the Van Hiele model such as comparing the number of equal sides. Mitchelmore and Outhred (2004, p.467) observed that this is often done in comparison with everyday objects; for example, a rectangle is formed in the mind because it looks like a box. Carraher, Nunes and Schliemann (1993) characterise this as a child making a link between the dichotomy of *school* and *street* mathematics. It could be conjectured that this formative geometric reasoning is normally only applied to *Euclidean spaces* (shapes or figures which a defined by a set of axioms or postulates) in the school environment but could be applied to objects in everyday life. Furthermore, the Van Hiele model (1985) does not acknowledge that children learn in a number of different ways; Baume and Fleming (2006, p.5) suggest children mostly learn through multiple representations of a problem. This and the success of using multisensory approaches in teaching may influence the rate of progression in children's geometrical knowledge.

French (2004) states that at primary school level, the teaching strategies used are often a mixture of inductive (practical investigations and kinaesthetic activities) and deductive (formal teaching and exposition) which constitute the first stage of Geometry teaching (Ofsted, 2012 a). There does seem to be evidence that children are influenced by deductive teaching, particularly in their approach to prototypical images.

Kerslake (1979, p.34) investigated whether primary-school-aged children could recognise angles and shapes of different orientations; she found that most pupils only correctly identified the 'typical' image (the orientation of angle/shape that was normally drawn by their class teacher) whereas 'atypical' images were not recognised. Burger (1986, p.41) rationalises this as younger children often believing a rule based on one example, normally from the class teacher, and being unable to extrapolate this to other shapes.

This research seems to validate Piaget's assertion that, at this stage, children view figures holistically without any realisation of their properties. However, Sperry's (1961, p.1750) hemispheric dominance theory suggests that children who have a natural ability in Geometry, normally those with a predisposition to the right cerebral hemisphere, may not be categorised by this developmental model. Carter (2004) disagrees with Sperry's theory and questions the validity of it. Although there may be some disagreement with the various cognition theories, it could possibly be assumed that only some children can make connections between shape at this stage of learning.

Upon learning about angles and lines, children may be able to make better links between shapes. Van Hiele (1985) termed this level as *analysis* where pupils could understand the properties of shape but not yet link them. Piaget (1967) and Bruner (1961) both support this in their respective *pre-operational stage* and *symbolic* models, although Bruner recognises that these seemingly autonomous mental structures can be blended together and related, depending on the age and experience of the child. This model is in sharp contrast to Piaget's age-centric theory; Bruner philosophises that a child can learn any task given the right teaching. Again, perhaps due to the abundance of command, teacher-led strategies (Mosston, 1966), pupils may develop an inflexible arbitrary knowledge of Geometry which can be a barrier to progression (Hewitt, 1999).

Once children have grasped the basic notions of angle and shapes, they can begin to make links between them. At the Piagetian *concrete operations* stage, a child can think logically and solve problems which are heavily generalised and require an inductive manner of thinking. Ofsted (2012) compare this to the situation in GCSE exams where children are generally able to competently solve 'method' questions but sometimes struggle with 'worded' or 'applied' problems.

Roughly when a child starts secondary school, they enter the Van Hiele *abstraction* stage where they can compare shapes and make connections between them such as in the diagram below:

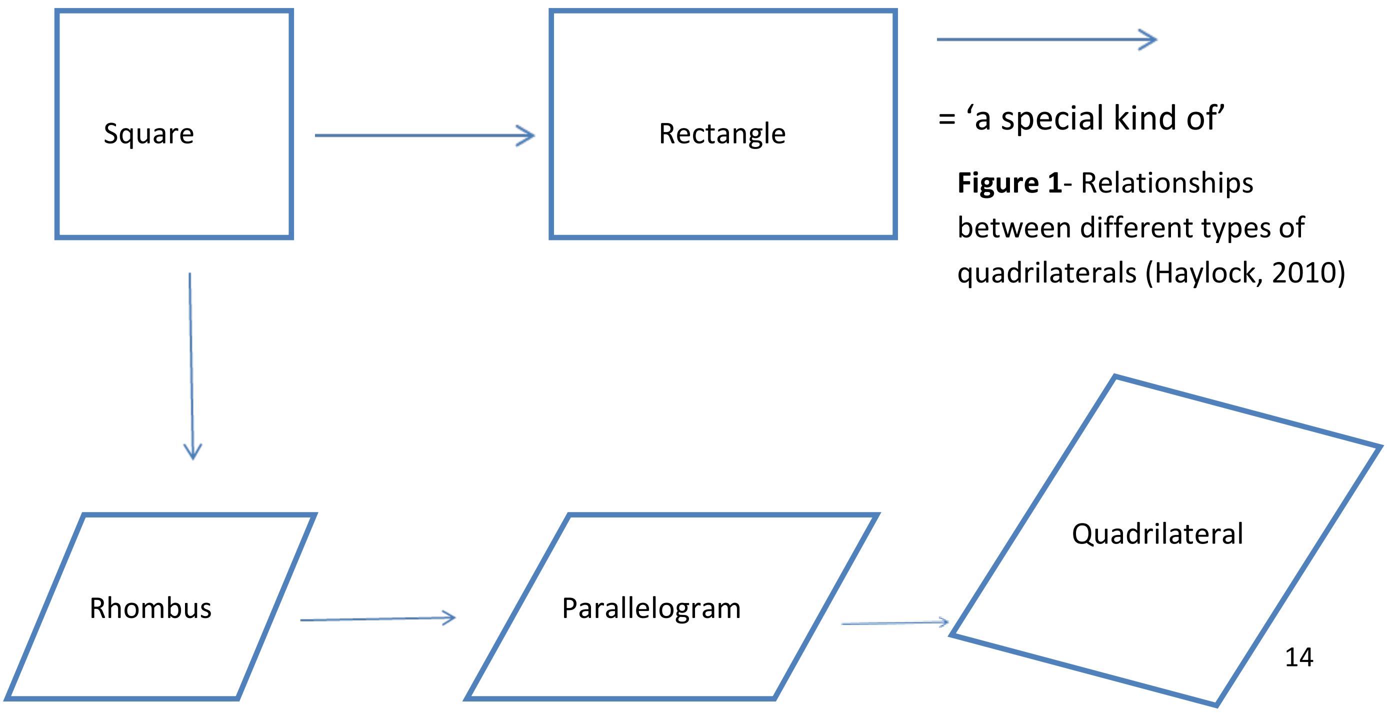

Figure 1- Relationships between different types of quadrilaterals (Haylock, 2010)

The progression to a child thinking in a slightly more abstract manner and knowledge of the properties of 2-D shapes may help a child to understand *plane Geometry* and that of 3-D *polyhedra* and *platonic solids* such as *cubes* and *tetrahedrons*. DfE (2013 d) highlight that a knowledge of Euclidean Geometry and a developing knowledge of spatial awareness (through studying topics like tessellations) is conducive to understanding *Affine Geometry*, the study of parallel lines which is introduced in Mathematics at KS3 Level.

The progressive and interrelating nature of Geometry seems to be further represented by the fact that knowledge of Affine Geometry can help with understanding of the beginnings of C*o-ordinate Geometry* in Secondary School in modules like transformations (DfE, 2013 d). It may also be beneficial to study *Vector Geometry*, both at secondary school (translations) and A-Level (magnitude, direction, scalar product and equation of a vector).

However, Haggerty (2001) asserts not all geometrical learning is linear and discrete; it can be discontinuous as pupils develop at different rates. A possible criticism of the Piagetian and Van Hiele models is that they are heavily generalised and do not account for variations in ability. Furthermore, Piaget's model is *domain specific* and surmises that cognitive development is *homogenous* across all fields which may not be true; as the Organisation for Economic Co-operation and Development's (OECD) (2008) distinction between *pure* and *applied* mathematicians implies.

It seems that a clear understanding of all the fields of Geometry is needed before a child can develop deductive logic and understand formal Euclidean proofs such as proving there are 180 degrees in a triangle. Piaget (1953) argues that children do not enter the *formal operational* stage until they are 14 and that they cannot learn formal proofs before this period. Van Hiele (1985) describes similar properties in his penultimate geometric level *deduction* although he does not specify which age pupils reach this level. This seems to be supported by the curriculum as the DfE (2011 b) states that proofs are not usually covered until Year 10 although some students study it in Year 9 in accelerated study programmes.

A supposition could be presumed that all students need a good comprehension of Algebra to comprehend more sophisticated Geometry topics. This seems to be evidenced by curriculum content; GCSE and A Level Mathematics contain more *Algebraic Geometry* and a reduced amount of Euclidean Geometry. Indeed, Geometry in A Level Mathematics is almost exclusively made up of Co-ordinate and *Differential Geometry* (Calculus) and some Trigonometry with barely any pure Euclidean Geometry. Parliament (2012) and Ofqual (2012) perceive this to be a weakness of the course, which could stop pupils reaching the final Van Hiele (1985) level of *rigour,* where a pupil has mastered all the axiomatic structures of Geometry and can confidently deal with Non-Euclidean Geometries.

In learning Geometry, pupils seem to develop from pure and synthetic Geometry (Euclidean) but need to have an understanding of Algebra to understand more sophisticated levels of analytic (Algebraic Geometry). There may be a *finite* level of geometrical reasoning that a student can reach and that their understanding of Geometry will eventually plateau.

Research carried out by Senk (1989, p.308) and Gutiérrez and Jaime (1998, p.37) describes the Van Hiele model positively and highlights its impact on the American Mathematics Curricula. However, Burger (1986, p.41) highlights a deficiency of the model as the levels of knowledge within it are discrete, not continuous, and in some cases overlap, with children sometimes displaying reasoning at numerous levels simultaneously. Conversely, this may actually be an asset of the theory: it could be more widely interpreted and thus may be applicable to more research studies.

The literature seems to suggest that determining whether the Van Hiele Model is appropriate in assessing children's geometrical abilities is something which needs to be examined. The potential impact of knowledge of the Van Hiele model may have on teaching and learning also seems to be a relevant issue to be considered.

Methodology

Throughout my teaching practice and career I have always tried to be a reflective practitioner and recognise what needs to be changed about my own and possibly whole school practice. Hubbard and Power (1999) and Bell (2005) argue that developing this thoughtful style may allow me to assess pupils more in-depth in this investigation to facilitate more accurate analysis and demonstrate good practice in conducting my research study.

My model of research is not essentially interactive as it is being individually conducted by me. Vygotsky (1978) argues this modality of inquiry may deprive me of the possible collaborative benefits of a *social constructivist* model of research. According to Freire’s (1982, p.30) theory, my style of action research is still *participatory* as I am trying to enforce change using a reflective approach, but only on an individual level. Dadds (1998; 2009) argues that trying to enforce change is a key attribute of practitioner research which is something I am trying to do in my study. Ollerton (2004) highlights this as the key distinction between a reflective practitioner and a practitioner researcher, actually doing something specific about the issue. However, I may be able to amalgamate the most desirable assets of both roles in my study by changing things but also being reflective in my practice.

I am using what I term a 'peflective' paradigm in my approach to this research study (Curran, 2013). 'Peflective' signifies for me that I am taking a positivist viewpoint with a reflective element.

The style of action research implemented in my study is very reflective and cyclical by identifying a relevant theory, collecting data with my tasks and reflecting *and* reacting to the research by changing mine and possibly influencing other teachers' strategies in teaching shape. However, McNiff and Whitehead (2002) highlight that there may only be limitations to what I can change, something which indicates I may need to keep improving and refining my practice.

I am using a *positivist* paradigm in my approach to this research study. One theory of how children learn Geometry, Van Hiele's (1985) model of geometric reasoning, is used to construct my research study. I am taking a 'realist' view of the classroom environment as I am summative assessing children's geometrical ability in my research study by analysing their scores using the Van Hiele model (1985) to produce quantitative data. Variables are to be very tightly controlled in this test, and I believe that I can control all of them adequately: the tests will take place in non-mathematical rooms at each school and each child will have access to the same equipment, instruction and resources. Hudson and Ozanne (1989, p.2) recognise that, although a positivist approach is logical and may produce objective data, it may not address the underlying cause and reasons for the data occurrence which an ‘idealist’ ontology might yield.

Carson et al. (2001) state that a researcher using a positivist ontology stays emotionally detached of the setting and research process. This is something I would like to create in my study. However, I will have to work hard to ensure this neutrality given that the research is being conducted in my former educational establishments, which may induce understandable emotional attachments.

The epistemological viewpoint I have taken is also positivist as I believe in quantifying intelligence through tests, measurement and observation. Conversely, Stenhouse (1974) is a proponent of the interpretivist approach which he feels has *rigour* as the power of research lies with the teacher. However, the potential weakness of a positivist methodology may be negated by the logical positivism that is used- the assessment made of the pupils is being made using a fairly reliable theoretical framework. Howson and Urbach (1993) advocate the credentials of logical empiricism, something which I have used as tasks 4 and 5 rely on the scientific verification of prototypical images which seems a reliable framework on which to base my conclusions on.

I have used a *mixed methods* paradigm in my collection of data. Johnson et al. (2007) define this as collecting a mixture of qualitative and quantitative data and using both viewpoints to justify my conclusion which is what I have tried to implement in this study. This is exemplified by the duality of my approach in analysing task 4 where participants are asked to draw a rectangle that looks visually appealing. Although the analysis is partially positivist in conducting a statistical test (non-parametric one sample t-test) and using known theoretical research (about the golden ratio), it is also interpretivist as the test used is *inferential* so a supposition about the data can be assumed. Furthermore, the results will be related to the literature review and my own observations to see how useful the Van Hiele Model is in assessing how pupils learn Geometry.

I have taken a *deductive* approach in writing my literature review (see below diagram) as I examined theoretical approaches in order to structure my approach but have implemented an *inductive* method of data collection as results are collected and then related to practice. The fusion of these 2 approaches may be complementary as it could allow me to gain a deep knowledge of what I have researched and enact what I have learned in my classroom practice (Weick, 1979).

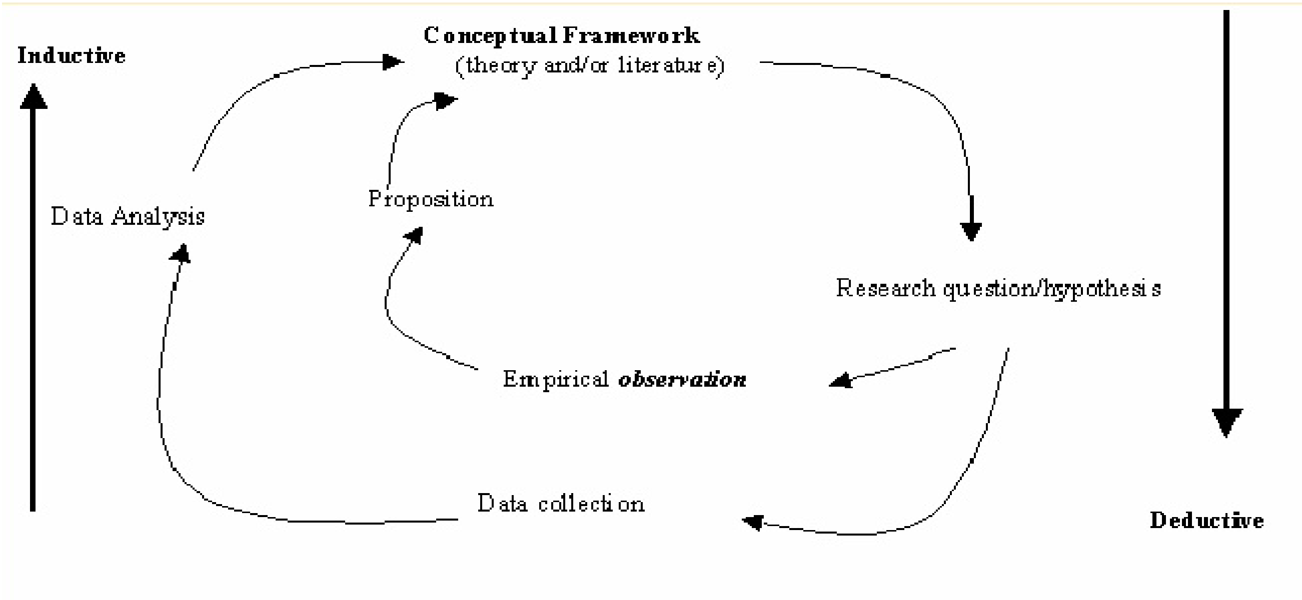

Figure 2- Inductive and Deductive research methods
(Newton and Rudestam, 2012)

In conclusion, the methodology I have used seems sound as it tries to complement a positivist research paradigm with a mixed methods paradigm in data collection.

Data Collection (See Appendix 2, p.60-61 for more detailed information)

The tests for my study consisted of 10 pupils completing 5 geometrical tasks (see Appendix 3, p.62-70 for a detailed explanation on them) and 37 teachers also completed a questionnaire (See Appendix 4, p.71-72) to ascertain their views on Geometry which was then compared with the data from the pupils. Likert (1932) proposed a standard 5-point answer scale for questionnaires. However, I chose a 4-point scale by eliminating the 'Neither Agree Nor Disagree' option so I could eliminate neutrality and gain a stronger polarity of opinion.

Sample

The data were collected at a primary and secondary school and a sixth form college in the same town all within a two mile radius in the North East of England.

A *teacher-selected, systematic sample* was used in the collection of the data. 2 pupils were tested from each Key Stage 1-5 in a *systematic* approach by taking the same number of pupils from each stage.

The sample size of 10 students may not be entirely statistically reliable. Bartlett, Kotrlik, and Higgins (2001) articulate larger sample sizes as being generally accepted to have increased precision and statistical power, whereas reduced samples tend to have decreased confidence intervals and a greater susceptibility to outliers. This seems to be evidenced by the dubiousness of whether the results of this study would be replicated in a larger investigation.

On the other hand, Haeussler, Paul and Wood (2013) advocate the advantages of a small sample size being *expedient* and *necessary* as it allows data to be collected and analysed efficiently although they recognise the potential limitations in accuracy a small sample size could have. The cognitive differences in the age of the subjects involved in the study seem to validate this: developmental differences seem far more prominent amongst children than adults (Piaget, 1952; 1953).

Theoretical Framework

All tests are analysed using the Van Hiele (1985) model of Geometrical reasoning (See Appendix 1, p.53-59 for a more detailed version).

Tests (See Appendix 3, p.62-70 for more detailed explanations of tests)

1. **Estimating length of line** **Van Hiele Level:** 2
2. **Estimating the size of an angle** **Van Hiele Level:** 2
3. **Draw a right angle** **Van Hiele Level:** 2
4. **Draw 4 different types of triangle** **Van Hiele Level: 2**
5. **Draw a rectangle that looks nice** **Van Hiele Level:** 3

Ethics

Throughout the research study, an ethical approach was followed at all times (See Appendix 5, p.73-74). Particularly due to the young age of the participants in the study, full school and parental consent was sought and obtained (See Appendix 6, p.75-76) and a transparent and safeguarding approach was followed at all times- schools were fully involved in the testing procedure; all appropriate protocol was followed and participation was entirely voluntary and the children and school had the right to withdraw at any time. Permission also needed to be gained for the possible publication of the study.

The ethnicity of most of the pupils at the schools surveyed was White British. However, all students were selected free of bias and no discrimination was made at all, particularly for cultural factors such as religion and race.

Furthermore, due to the importance of the study, it was ensured that the benefits were reciprocal and that the research was challenging. The schools will have a more detailed knowledge about how their pupils learn Geometry and the study will influence the author’s and other practitioners’ teaching strategies when teaching shape. In addition, an original approach has been followed as the study is completely of my own design and examines a field which has not been extensively researched as other areas of Mathematics.

Results (See Appendix 7, p.77-82 for raw data of investigation)

1. **Estimating Length of Line**

Key Stage	Van Hiele Task Level	Demonstrated Van Hiele Level of Child 1 (Male)	Demonstrated Van Hiele Level of Child 2 (Female)
1	2	1	1
2	2	1	1
3	2	3	2
4	2	4	2/3
5	2	2	3/4

The criteria for satisfying each Van Hiele Level are shown below:

Van Hiele Level	Criteria
1	Basic Guess with no real method of how to estimate line. Far away from true value.
2	Simple, logical method of method of guessing length i.e. dividing into 1 cm segments. Quite close to true value.
3	Attempting to picture and visualise line as part of a 2-D shape (i.e. square or rectangle) and possibly drawing that shape down and estimating length of line from that. Close to true value.
4	Estimating length of line through proofs using 2-D shape such as a Circle (Circle theorems). Very close to true value.
5	5- Estimating length of line by splitting it into golden sections or using 3-D proof system. Exact or nominal distance away from true value.

2. **Estimating the size of an angle**

Key Stage	**Van Hiele Task Level**	**Demonstrated Van Hiele Level of Child 1 (Male)**	**Demonstrated Van Hiele Level of Child 2 (Female)**
1	2	1	1
2	2	2	2
3	2	3	1/2
4	2	3	3/4
5	2	3	3

Criteria for reaching each Van Hiele Level:

Van Hiele Level	**Criteria**
1	Recognising it is an angle but not sure on how to quantify it. Basic Guess. Far away from true value.
2	Recognising it is an obtuse angle and must be between 90 and 180 degrees. Quite close to true value.
3	Attempt to divide the angle up into constitute angles (i.e. 90° + 30°) and by drawing the angle in a 2-D shape such as rectangle or parallelogram. Close to true value.
4	Estimates size of angle using affine Geometry proofs (i.e. supplementary angles add up to 180°, corresponding and vertically opposite angles equal). Very close to true value.
5	5- Uses 3-D vector notation to estimate size of angle. Exact or nominal distance from true value.

3. **Draw a right angle**

Key Stage	Van Hiele Task Level	Demonstrated Van Hiele Level of Child 1 (Male)	Demonstrated Van Hiele level of Child 2 (Female)
1	2	1	1
2	2	2	2
3	2	2	2
4	2	2	2
5	2	2	2

Criteria for reaching each Van Hiele Level:

Van Hiele Level	Criteria
1	Need intensive support on understanding right angle term.
2	Able to draw right angle of a prototypical orientation without any prompts, possibly as part of a 2/3D shape.
3	Draws a right angle of a non-prototypical orientation, possibly as part of a 2/3D shape.
4	Draws a right angle using some sort of simple Geometric proof: i.e. Angle in a Semicircle will always equal 90°.
5	Draws a right angle using Non-Euclidean Geometry i.e. Elliptic or Hyperbolic.

4. Draw different types (right angled, equilateral, scalene and isosceles) of triangle

Key Stage	Van Hiele Task Level	Demonstrated Van Hiele level of Child 1 (Male)	Demonstrated Van Hiele level of Child 2 (Female)
1	2	1	1
2	2	4	4
3	2	3	2
4	2	4	4
5	2	2	3

Criteria for understanding of task and shapes:

Grading	Criteria
1	Intensive support needed to understand task.
2	Able to draw some triangles with prompts.
3	Able to draw all triangles correctly with some prompts.
4	Drew all triangles correctly with no prompts.
5	Used extra mathematical notation such as equivalence signs.

Criteria for reaching each Van Hiele Level:

Van Hiele Level	Criteria
1	Has knowledge of what a triangle is but cannot distinguish between the different types and needs heavy prompting.
2	Student can draw all the triangles accurately without/ with some prompting and has quite a clear knowledge about the differences between the types of triangle. Triangles are likely to be of a prototypical orientation.
3	Clear and solid definitions of different types of triangles, possibly with some non-prototypical orientations.
4	Congruency and equivalence acknowledged with use of appropriate symbols. May draw triangle used in formal proofs such as isosceles triangle formed by 2 radii or triangle in a semicircle is always a right-angled one.
5	Recognising triangles in other axiomatic systems such as Hyberbolic and other Non-Euclidean Geometries.

5. **Draw different types (right angled, equilateral, scalene and isosceles) of triangle**

Key Stage	Van Hiele Task Level	Demonstrated Van Hiele level of Child 1 (Male)	Demonstrated Van Hiele level of Child 2 (Female)
1	2	1	1
2	2	4	4
3	2	3	2
4	2	4	4
5	2	2	3

Criteria for understanding of task and shapes:

Grading	Criteria
1	Intensive support needed to understand task.
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4	Drew all triangles correctly with no prompts.
5	Used extra mathematical notation such as equivalence signs.

Criteria for reaching each Van Hiele Level:

Van Hiele Level	Criteria
1	Has knowledge of what a triangle is but cannot distinguish between the different types and needs heavy prompting.
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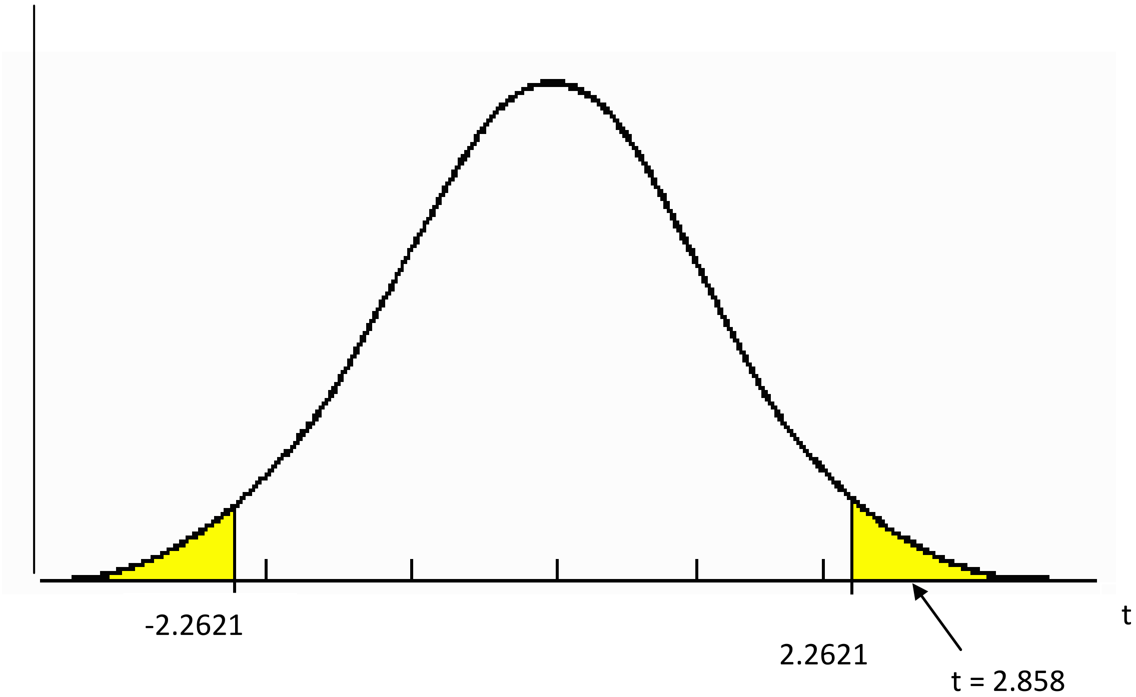
3	Clear and solid definitions of different types of triangles, possibly with some non-prototypical orientations.
4	Congruency and equivalence acknowledged with use of appropriate symbols. May draw triangle used in formal proofs such as isosceles triangle formed by 2 radii or triangle in a semicircle is always a right-angled one.
5	Recognising triangles in other axiomatic systems such as Hyberbolic and other Non-Euclidean Geometries.

Task 5 T-Test Analysis

Research Hypotheses

H_0= {μ=1.618 (φ) ; The average ratio of all participants’ rectangles dimensions will equal the golden ratio.}

H_1= {μ ≠ 1.618 (φ); The average ratio of all participants’ rectangles dimensions will not equal the golden ratio.}($t = 2.858$, $df= 9$, $p < 0.05$)



The critical value of t is +/- 2.2621: so the t-value must be either less than -2.2621 or greater than 2.2621 to be significant at the 95% confidence level.

The t-value of the pupil’s rectangles is 2.858 (See Appendix 8, p.83 for full statistical data). From this, we can reject the null hypothesis at the 95% confidence level and state that students do draw rectangles with dimensions that have different ratios to that of a golden rectangle.

Summative Table of Van Hiele Levels Demonstrated by pupils

Task	KS1		KS2		KS3		KS4		KS5	
	M	F	M	F	M	F	M	F	M	F
1										
2										
3										
4										
5										

Key

Below Van Hiele task level

Same as Van Hiele task level

Above Van Hiele task level

Questionnaire Analysis

No. of teachers surveyed: 37 (Primary: 26, Secondary: 6 and Sixth form: 5)

Question	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
1. I enjoy teaching Geometry	0	3	29	5
2. There is enough Geometry in the Curriculum	0	1	33	3
3. I teach Geometry more than other areas of Mathematics	13	23	1	0
4. My pupils understand Geometry better than other areas of Mathematics*	0	29	6	0
5. Children enjoy Geometry more than other areas of Mathematics^	0	21	14	0
6. Geometry is a quite complex topic to teach	0	21	15	1
7. Most of my Geometry lessons are practical	0	6	26	5
8. Geometry is more important than other areas of Mathematics	5	21	7	4
9. I feel there should be more guidance and theory on how to teach Geometry#	1	23	9	3

* 2 teachers wrote 'It depends on the child'.
^ 1 teacher wrote 'It depends on the child' and one did not answer this question.
# 1 teacher did not answer this question.

The data seem to suggest that, whilst most teachers enjoy teaching Geometry, they do not teach it very often. Although the teachers sampled felt that Geometry was easy to teach, there was a contradiction in that the majority thought that their pupils do not understand or appreciate Geometry as much as they do other areas of Mathematics. However, this is only a tenuous conclusion, and needs a much larger sample to even tentatively validate this statement.

Question 10 Rectangle Analysis

Primary School

Number of teachers surveyed	Average Longest Side (D_1)	Average Shortest Side (D_2)	D_1/D_2
26	5.9	2.8	2.11

Number of pupils surveyed	Average Longest Side (D_1)	Average Shortest Side (D_2)	D_1/D_2
4	9.18	3.18	2.89

Secondary School

Number of teachers surveyed	Average Longest Side (D_1)	Average Shortest Side (D_2)	D_1/D_2
6	6.05	3.27	1.85

Number of pupils surveyed	Average Longest Side (D_1)	Average Shortest Side (D_2)	D_1/D_2
4	10.08	3.88	2.59

Sixth Form

Number of teachers surveyed	Average Longest Side (D_1)	Average Shortest Side (D_2)	D_1/D_2
5	6.52	2.64	2.47

Number of pupils surveyed	Average Longest Side (D_1)	Average Shortest Side (D_2)	D_1/D_2
2	8.5	6	1.42

Interestingly, there seems to be hardly any correlation between the teachers' and pupils' results, although the teachers' results are much more clustered than the pupils'.

Discussions of Results

A striking feature of the study was the prototypical orientations of the shapes that students drew. All the shapes drawn by the pupils were prototypical with the exception of one female KS3 pupil (See Appendix 9, p.84) who drew her triangles in task 5 with a non-prototypical orientation. However, upon being asked if a rotated rectangle was still the same shape (See Appendix 10, p.85), pupils replied correctly that it was. This would seem to give credence to the concept of prototypical images; students copied shapes they remembered teachers/tutors drawing on the board or diagrams they have seen a book, which all seem to be prototypical. In task 4, the KS4 pupils drew rectangles that were almost congruent (See Appendix 11, p.86-87). From discourse analysis, both stated they remembered a teacher drawing a similar rectangle in their exposition. The heavy influence of a teacher seems to be further backed up by the rectangles that were drawn in the teacher questionnaire, which were exclusively prototypical in orientation.

As aforementioned, Carraher, Nunes and Schliemann (1993) disseminate mathematics into *school* and *street* classifications. They argue that sometimes teachers do not acknowledge how children learn Geometry and any innate spatial abilities they have. This investigation seems to validate that, in Euclidean Geometry; children are mainly presented with shapes of a prototypical orientation. This is *despite* some pupils having considerable geometrical acumen; most pupils in this study knew that if a shape or angle was rotated, it was still the same. From this, 2 main implications for my practice seem to arise.

Firstly, if I were to draw non-prototypical orientations of shape in their Geometry lessons, this may help more pupils recognise unfamiliar shapes and become more adaptable in their mathematics learning. Furthermore, more varied examples may engage students more in the subject matter. At a higher level, it could help them to cope better with the unfamiliar 'worded' questions which are becoming increasingly popular in mathematics exams (DfE, 2012).

Secondly, if I recognised the unique ways in how students learn Geometry (i.e. by interacting with the world around them), this may help them to facilitate a more personalised style of learning. This could result in a more student-directed lesson where teachers encourage pupils to give their own geometrical examples (Mosston, 1966). Coupled with an interactive style of teaching style, this could increase an individual's motivation and resilience as they may feel a sense of belonging and worth if they see that their ideas are valued

This could also give other pupils access to the fascinating innate spatial skills children already have. For example, in task 5, a male KS1 pupil drew 4 *similar* isosceles triangles (See Appendix 12, p.88). In addition, a female KS1 pupil drew 2 congruent scalene triangles (See Appendix 13, p.89). Despite being at such a formative stage of their education, the pupils had knowledge of sophisticated geometrical concepts. The fact they probably did it subconsciously seems to give further credibility that some children have considerable innate spatial ability. If unlocked by the teacher, this could lead to real progress in geometrical learning.

In addition, another implication on my practice from the study is my teaching style. A more interactive teaching style in Geometry lessons could allow students to progress further in the topic. This seems to be backed up by both theory and practice: DfE (2009; 2012) articulates the importance of social learning in the classroom, particularly in terms of the teacher's orientation, and many of the pupils questioned in my study said they preferred learning Geometry in this way as they got more out of it and saw it as a break from the 'usual Mathematics lessons'. This teaching strategy seems to be something I can incorporate into all my Mathematics lessons, although it may be even more pertinent to do so in Geometry lessons.

Although the conclusions of my study may be valid, a larger scale study would probably have yielded more accurate results. Furthermore, the practicalities of implementing such a personalised model of learning need to be considered.

This was one of the main problems of my study: the scale. If a larger size sample was collected, then the results may be more reliable and slightly more concrete points could be made. However, the intimacy of the sample allowed for more careful and intricate analysis, which yielded some interesting points.

Indeed, I was very pleased with how I conducted the study. All procedures followed were fully ethical and the schools and pupils were genuinely interested in my research. My research methods were consistent and the rigid positivist ontology used in the study complemented the more flexible mixed methods paradigm utilised in the collection and analysis of the data. A way of improving my study could have been to give the pupils an alternative form of the questionnaire distributed to the teachers so the 2 sets of responses could be cross-referenced.

The questionnaire analysis exemplified this advantage of my study as it allowed me to compare the primary, secondary and A level Mathematics teachers’ abilities and viewpoints on Geometry. Interestingly, the average ratios of the dimensions of the primary school teachers’ rectangles were closer to the golden ratio than that of the sixth form teachers. This is despite some of the sixth form teachers pre-guessing that this task was to do with the golden ratio. One variable that could have influenced the teachers’ drawings was the box in which they constructed the rectangle; the box measured 12.2 cm by 18.5 cm. Dividing the longest side by the shortest side gives an answer of 1.52 (2 d.p.) which is not dissimilar to the golden ratio of 1.618. In a similar manner to children interpreting prototypical images, this could have led the teachers to subconsciously scale their rectangles to be similar in proportion to the box, which could explain the relatively uniform spread of the data around the Golden Ratio. Interestingly, 2 primary school teachers drew an almost congruent rectangle to that of the box, in both size and proportion (See Appendix 14, p.90-91).

It could be surmised that, as the primary school teachers have more pedagogical interactions with Geometry than sixth form teachers, then this is the reason why their rectangles were closer to the golden ratio, simply because they have more experience of *drawing them*. The same could be said of the secondary school teachers, as they obviously teach Geometry on a regular basis, particularly Euclidean Geometry. The notable feature of their rectangles was that they were precise in their drawing of them, whilst some used extra mathematical notation by labelling right angles and parallel and sides of the same length (See Appendix 15, p.92-93).

Whilst the secondary school teachers did draw rectangles that were closer to the golden ratio than primary schools, their sample size was small and their accuracy may not be replicated over a larger sample. In addition, primary school teachers were more adept at recognising that a rectangle could have different orientations; of the 5 secondary school teachers sampled, 4 drew a rectangle of the same orientation, despite them being sampled independently. It could be conjectured that, whilst secondary school teachers have a more *in-depth* knowledge of Geometry than primary school teachers, primary school teachers have a more *flexible* grasp of Geometry than secondary school teachers, as shown by their appreciation of the various orientations of a rectangle.

The Van Hiele model was a useful theory on which to base my theory. It related very well to my study and was relatively proficient in helping giving me practical ideas to improve my teaching. However, it was more useful in assessing how good the pupils were at Geometry because I was able to place them on a levelled numerical scale, which complemented my positivist approach quite well. However, in some cases it was difficult to grade the pupils and I gave a small minority of pupils a score which was halfway between two levels. A more flexible model may have been more beneficial. Nevertheless, it helped me establish some useful conclusions.

Perhaps unsurprisingly, there was a trend of older pupils achieving a higher Van Hiele Level in the tasks than their younger counterparts, due to their greater experience of learning Geometry. However, on some tasks, younger students were stronger than older students. For example, in task 5 (‘Draw 4 different types of triangle’), the male KS5 pupil attained an inferior Van Hiele level than the KS2 pupils. The KS2 pupils were able to accurately identify and draw all 4 different types of triangles, even spelling them (See Appendix 16, p.94-95). Conversely, the KS5 pupil took quite a lot of time to complete the task and only drew 3 different types of triangle, as he did not remember to draw a scalene triangle. He did, however, recognise the duality of a triangle in that it can be both isosceles and right-angled which compared unfavourably to the Female KS5 pupil’s level 4 Van Hiele score in this task (See Appendix 17, p.96-97). His slight deficiency in geometrical knowledge could be due to the surfeit of shape topics at A Level.

The task that pupils scored lowest on was task 3 (‘Draw a right angle’) with no-one achieving above Van Hiele Level 2 in this task. As there is little in the Mathematics curricula on right angles that exceeds Van Hiele level 2 in complexity, it may have been unjust to expect pupils to attain a high level in this task. Furthermore, the criteria in reaching certain Van Hiele levels in this task were fairly demanding, which contributed to the pupils’ relatively low scores in this task. However, for the most part, the results I obtained correlated with theory: no pupil scored above level 4 in the study which the model says that children will reach upon leaving secondary school (Van Hiele, 1985) and most scored around a level 2 in the Van Hiele model.

In conclusion, the Van Hiele model does seem a fairly reliable model to assess a children’s geometrical knowledge. Most children achieved a consistent range of scores and no child achieved above a Level 4 in the tasks, which corresponds with the theory. On a personal level, the Van Hiele model was very effective in increasing my knowledge of how children learn geometry and it has vastly improved my subject knowledge of Geometry. Having previously been uncertain of shape prior to conducting this study, I now feel much more confident in the subject matter. This will impact on my practice as I will give credence to the geometrical ideas that pupils generate and ensure that I will draw shapes of non-prototypical orientations in my Geometry teaching. Moreover, I feel this could be applied to every topic I teach in Mathematics; I will give alternative explanations of the topic, to satisfy the needs of learners and prepare them for the unfamiliarity of certain questions. Above all, I will follow the proverb; ‘It’s not how intelligent a child is, it’s what makes a child intelligent that’s important.’

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List of appendices

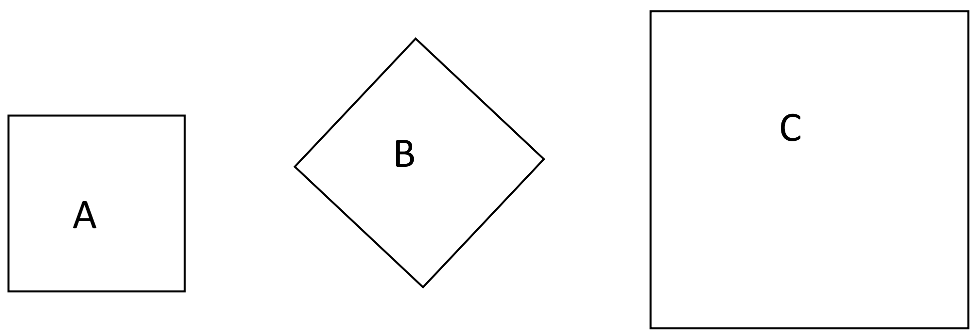
Appendix 1- Van Hiele Levels of Geometric Reasoning

(Van Hiele, 1985)

Level 1 (Visualisation)

At this initial stage, children are only aware of space as something that exists around them. Geometric concepts are viewed as total entities rather than as having components or attributes.

Children identify prototypes of basic geometrical figures (triangle, circle, square which are then used to identify other shapes in relation with everyday objects. A shape is a circle because it looks like a sun; a shape is a rectangle because it looks like a door or a box. A child at this level cannot see the similarities between a square and a rectangle or a rhombus and other parallelograms, so these shapes are classified completely separately in the child's mind.



A student at this level may recognise that shapes A and C as squares but might not appreciate shape B as one.

Level 2 (Analysis)

At this level, a student uses an informal analysis of a shape’s properties. Necessary properties of the concept are established. The objects of thought are classes of shapes, which the child has learned to analyse as having properties. A child at this level may state that a square has:

- 4 equal sides
- 4 equal angles.
- Its diagonals are perpendicular
- The diagonals also bisect each other.

The properties are more important than the appearance of the shape. If a figure is sketched on the blackboard and the teacher claims it is intended to have congruent sides and angles, the students accept this prototypical image without challenge. Properties are not yet ordered at this level.

Children can discuss the properties of basic Euclidean figures and recognize them by these properties, but generally do not allow categories to overlap because they understand each property in isolation from the others. For example, they will still insist that a square is not a rectangle. They may introduce extraneous properties to support such beliefs, such as defining a rectangle as a shape with one pair of sides longer than the other pair of sides.

Children begin to notice many properties of shapes, but do not see the relationships between the properties; therefore they cannot reduce the list of properties to a concise definition with necessary and sufficient conditions. They usually reason inductively (informally) from several examples, but cannot yet reason deductively because they do not understand how the properties of shapes are related and thus cannot grasp geometric proofs at this level.

Level 3 (Abstraction)

A pupil at this level of geometric thought can logically order the properties of shapes, form abstract definitions, and can distinguish between the importances of certain properties of a shape. At this level, properties are ordered. The objects of thought are geometric properties, which the student has learned to connect deductively. The student understands that properties are related and one set of properties may imply another property.

Students can reason with simple arguments about geometric figures. A student at this level might say, Isosceles triangles are symmetric, so their base angles must be equal. Learners recognize the relationships between types of shapes. They recognize that all squares are rectangles, but not all rectangles are squares, and they understand why squares are a type of rectangle based on an understanding of the properties of each. They can tell whether it is possible or not to have a rectangle that is, for example, also a rhombus.

An example of an interrelationship that students at this level would make, understanding that the sum of the interior angles in a pentagon is 3 times and hexagon 4 times the amount of the interior angles in a triangle is shown below:

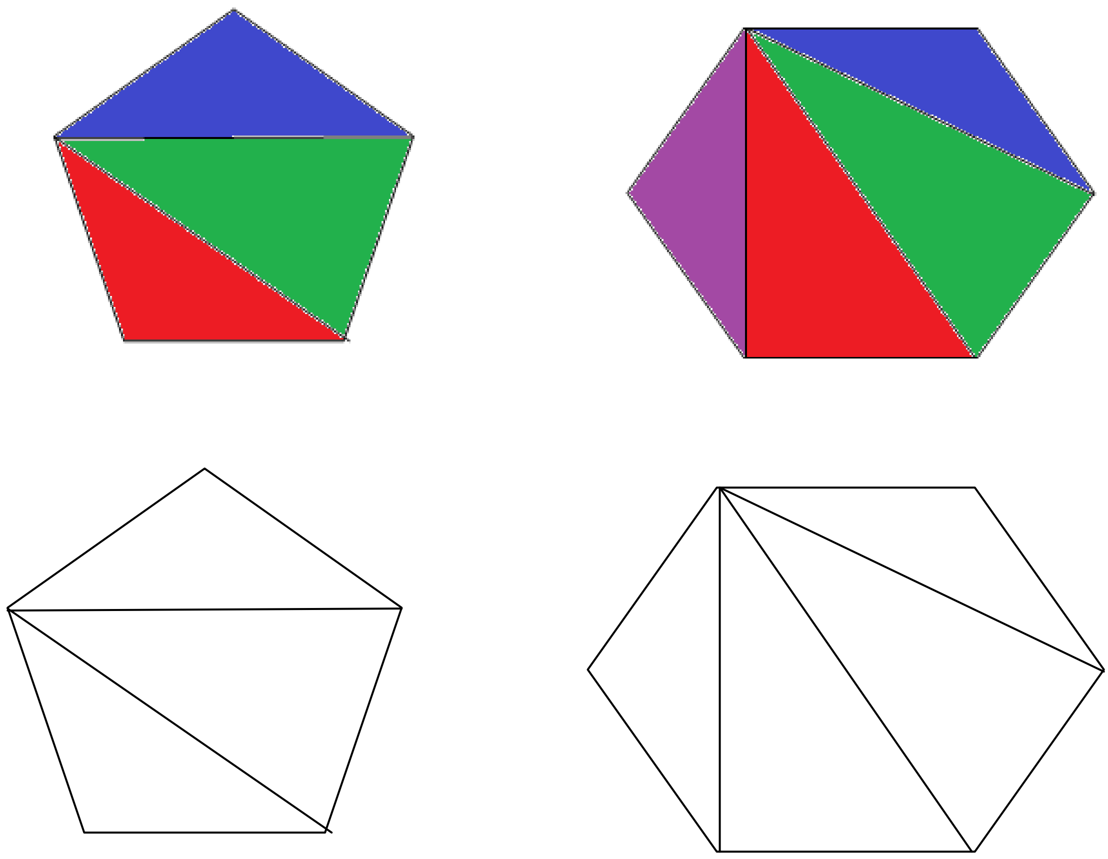
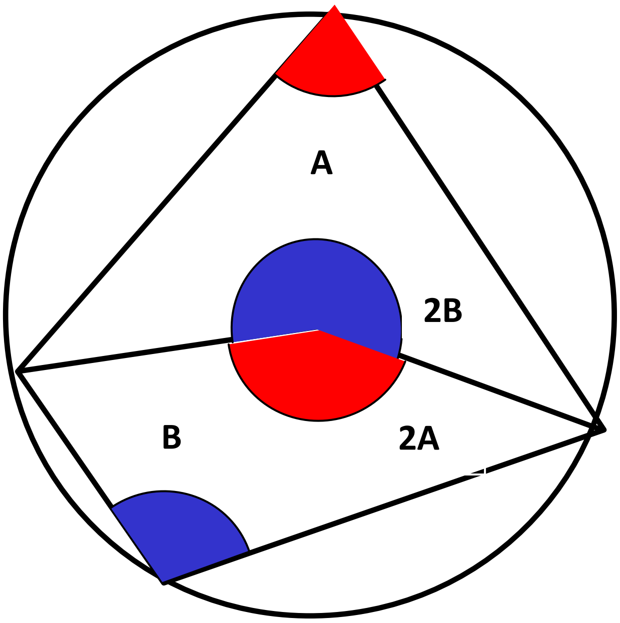


Figure 3- Pentagon and Hexagon Triangle Relationship

They understand necessary and sufficient conditions and can write concise definitions. However, they do not yet understand the intrinsic meaning of deduction. They cannot follow a complex argument, understand the place of definitions, or grasp the need for axioms, so they cannot yet understand the role of formal geometric proofs, although they may understand some informal proofs.

Level 4 (Deduction)

At this level, a person reasons formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions, and theorems. Students at this level understand the meaning of deduction. The object of thought is deductive reasoning (simple proofs), which the student learns to combine to form a system of formal proofs (Euclidean Geometry). Pupils working at this level can construct geometric proofs at a secondary school level and understand their meaning. They understand the role of undefined terms, definitions, axioms and theorems in Euclidean Geometry. However, students at this level believe that axioms and definitions are fixed, rather than arbitrary, so they cannot yet conceive of non-Euclidean Geometry such as Elliptic Geometry and Vector Geometry. Geometric ideas are still understood as objects in the Euclidean plane.



Angles at a point add up to 360.

Angle at centre is twice the angle at edge

∴ 2B + 2A = 360°

∴ 2 (B + A) = 180°

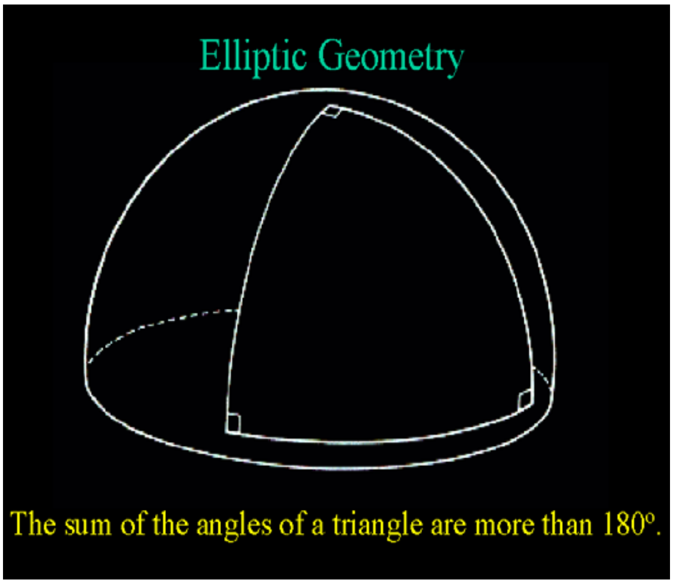
=> Opposite angles of a cyclic quadrilateral add up to 180°

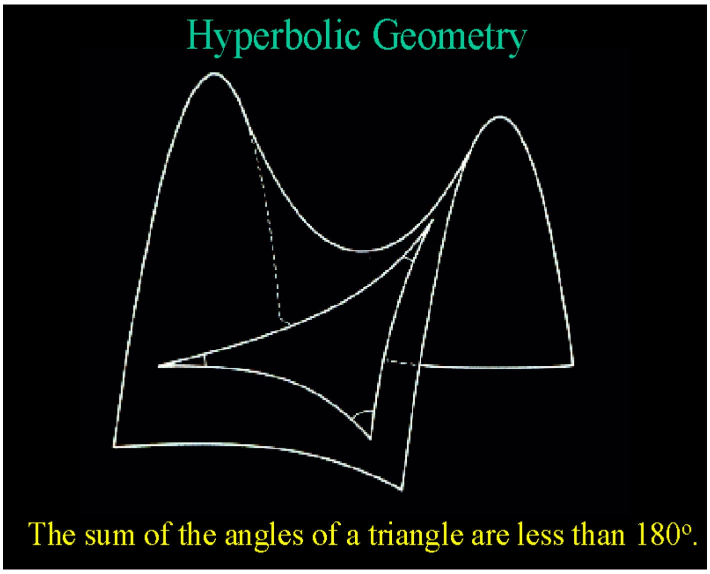
Figure 4- Cyclic Quadrilateral Circle Theorem Diagram

Level 5 (Rigor)

A student at this level can compare systems based on different axioms and can study various geometries in the absence of concrete models. This is the level of University level Mathematics and the level at which Mathematicians who specialise in Geometry work at.

People working at the *rigor* level can understand and work well with Non-Euclidean Geometries such as *elliptic* and *hyperbolic* Geometry.





Examples of proofs that people working at this level would understand.

Appendix 2- Full explanation of data collection

A *teacher-selected, systematic sample* was used in the collection of the data. However, the strategy used was partially *quota sampling,* which collects a set amount of individuals from a subgroup(s) (Graham, 2006), as the research necessitated an even number of males and females in the sample. It was thought that this would facilitate an easier gender comparison. The uniform nature of the data in using an even number of males and females in part represents a *stratified sampling* strategy as all of the schools surveyed had a roughly even gender split as the table below shows:

Type of School	No. of Pupils	No. of Males	No. of Females	Male: Female Ratio
Primary	364	193	171	53:47
Secondary	1219	602	617	49:51
Sixth Form College	2100	1063	1037	51:49

Figure 5- Gender data of schools involved in the study (DfE, 2013)

Having already ascertained a feature of the population in that it was *homogenous* in terms of gender, the sampling strategy was adjusted to be *representative* of the data and reflect its characteristics. The *dichotomous* nature of the variables sampled (male and female) allowed for easy stratification. Perhaps the only weakness of the method is that it generalises the data slightly.

In addition, there were also statistical advantages. A one-sample t-test was used to analyse the results of task 4 (comparing the dimensions of participants' rectangles against given mean of the golden ratio 1.618) which was appropriate as the empirical population statistics and variance were unknown. The t-test is 2-tailed; it acknowledges that the average of the ratios of the rectangles that pupils may be less or greater than 1.618. However, this analysis may not be *robust*- due to the high confidence intervals, potential skewness and variability of the data. A larger sample size would almost certainly have increased the statistical power. In addition, the assumption made in conducting a t-test that the data will closely conform to a normal distribution seems flawed. The cognitive differences in the age of the subjects involved in the study seem to validate this: developmental differences seem far more prominent amongst children than adults (Piaget, 1952; 1953).

Appendix 3- Full Explanation of tasks

Note: The KS3 Mathematics national Curriculum Attainment descriptors that the task corresponds to are in brackets

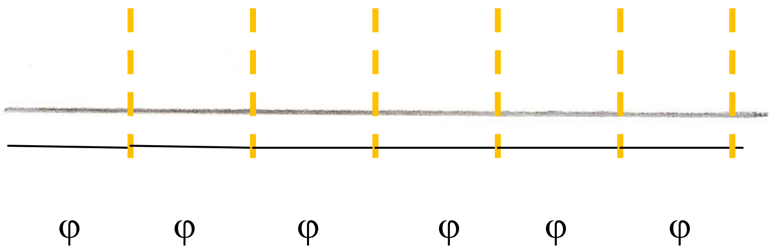
1. Estimating length of line (3.2 e - similarity, including the use of scale)

Although this is a fairly basic task, this still relies on children knowing the properties and classifications of shape which make it a Van Hiele level 2 task. However, students may demonstrate Van Hiele ability beyond Level 2 in this task if they use more sophisticated methods such as scaling and visualising lines on a shape.

The line will measure 10 cm. This seems an appropriate choice given that it is a fundamental number in Mathematics in systems such as place value and logarithms. It may also be a more accessible length for primary school children doing this task as this is a measurement they will have worked with quite often at school (DfE, 2011).

The line will be presented to pupils on A4 paper on the centre of the page. It is drawn in pencil of medium thickness. This is shown to pupils who will make a guess of how long the line is using the method they feel appropriate. They are allowed objects to help them to do this, with the exception of measuring instruments with a numerical scale.

Students will be given a varying amount of help depending on how they progress in the task. The support structure given may help to differentiate the Van Hiele level students are working at in estimating the line. If KS1 and KS2 pupils are struggling, they will be given objects such as pencils and books to help them. This may indicate they are at a lower level. Conversely, older pupils may not need as much help and could use more sophisticated methods of estimation. They may use a scaling structure, possibly by dividing it into unitary parts or at higher levels segmenting the line into *golden sections* (segments which measure 1.618 cm the golden ratio). The measurement was deliberately picked to induce this: as it is roughly 6 times the length of the golden ratio:



The participant's guess of the angle and distance away from the line (positive or negative, *bipolar*) will be calculated.

Students will not be able to see other participants' guesses.

2. Estimating the size of an angle (3.2 a - properties of 2D and 3D shapes, 3.2 e - similarity, including the use of scale and 3.2 f - points, lines and shapes in 2D coordinate systems)

Although the skills needed to do this task are fairly simple; to score accurately in this task a student should be working at L2 in the Van Hiele Model as they need to identify 'classes of angles'. Students who achieve a higher Van Hiele may have an extremely accurate guess or use more advanced methods such as dividing the angle up or using proofs.

The angle will measure 120 degrees. This may be a suitable choice due to its divisors and multiples. It has factors of 60 and 30, common angles students may recognise, and is also a third of 360 degrees, which is the known as arbitrary convention of a 'full turn', particularly in primary school mathematics (DfE, 2011). This may make it accessible to students of all ages- although its commonality may allow a student to recognise it instantly as a 120 degree angle.

The angle will be presented to pupils on A4 paper on the centre of the page. It is drawn in pencil of a medium thickness. Pupils will then estimate the size of the angle although they are not allowed to use geometrical equipment such as protractors. Also the research will take place in a room where there is no mathematical paraphernalia visible so the accuracy of the results is not compromised.

Students will receive scaffolding as appropriate to their level of comprehension of the task. Instructions may be repeated and if students are struggling to quantify the size of the angle, then I will remind them of the concept of a 'full', 'quarter' or 'half' turn without mentioning the number of degrees to ensure reliability.

Students' guesses will be recorded and the *bipolar* distance away from the true value of the angle will be calculated.

Students will not be able to see other pupils' guesses.

3. Draw a right angle (3.2 a- properties of 2D and 3D shapes, 3.2b- constructions, loci and bearings)

Students need to know what a right angle is to complete this task (although support will be given to those who do not) which requires some knowledge of the properties of shape. Students who are working at more advanced Van Hiele levels in this task may recognise that a 90 degree angle can have different orientations.

The purpose of this task is to investigate the orientation of how pupils draw a right angle. Most pupils should know what a right angle is and to remain objective and achieve an unbiased result.

Pupils will be presented with a sheet of A4 paper and asked to draw a right angle.

Some pupils may have the misconception of what a right angle is. I hope to correct this by scaffolding- instead of telling them it is a 90 degree angle I will use a real life example in the classroom which may appeal to their *imagination* and *visualisation* (Van Hiele, 1985).

Students' angles will be measured on their orientation away from a completely prototypical right angle.

Students will not be able to see other pupils' guesses.

4. Draw 4 different types (right angled, equilateral, scalene and isosceles) of triangle (3.2 a- properties of 2D and 3D shapes, 3.2 e- similarity, including the use of scale, 3.2 f- points, lines and shapes in 2D coordinate systems and 3.2 h- perimeters, areas, surface areas and volumes)

This requires pupils to classify triangles and they must have a good knowledge of the properties to do so. Students who are working at a lower Van Hiele level may not be aware of the differences between the types of triangle whereas pupils working at a higher level will understand more complex ideas such as equivalency and congruency.

The focus of this task is the level of sophisticated knowledge pupils have of *Euclidean figures* (shapes such as squares, rectangles and triangles) and the way in which students draw the shapes. Observation will be the main method of data collection utilised for this task. A predefined checklist of characteristics will be included as part of my observations which rates pupils on a numerical scale from 1 to 5. Although this may aid numerical analysis, this could mean that other incidents are overlooked, although an extra 'notes' column has been implemented to try and negate this (Lawton, 2008).

Students will be given A4 paper and a pencil to complete this task. Size is a fairly redundant variable in this experiment although it is still valid; the key attributes of the shape’s construction that will be analysed are its properties (size and angle). Pupils will be told what the 4 types of triangle are.

Instruction will be differentiated carefully to students who require a little help with the task. Only careful prompting will be given: I may remind them of a definition but I will not draw any examples for them, it has to be the children’s own research.

Students will not be able to see previous participants’ results or drawings.

4. Draw a rectangle that likes 'nice'/ 'attractive/ 'visually appealing' (3.2a- properties of 2D and 3D shapes, 3.2 e- similarity, including the use of scale, 3.2 f- points, lines and shapes in 2D coordinate systems and 3.2 h- perimeters, areas, surface areas and volumes)

It is quite subjective what level this Van Hiele task satisfies, but to carry out this task 'correctly' a student must have an inherent and detailed knowledge of shapes and have a lot of experience in viewing rectangles to understand the 'beauty' element of them. Children who are working at lower Van Hiele levels in this task may just draw a rectangle with no consideration of the dimensions whereas pupils who are working at advanced levels will think out a careful strategy possibly by using Geometric theories.

Minimal support will be given on this task so the result is unbiased. Children will be told to draw a 'nice-looking' rectangle on a piece of A4 paper which is provided. No information will be given on what I am looking for or expecting- it is entirely the child's own judgement of what a nice looking rectangle looks like. A ruler will also be provided to help them complete the task.

The only differentiation that is made in this methodology will be the adjective used in the instruction.

‘Nice- looking’ will be used for younger pupils (KS1 and KS2) as this is lexis that they will be able to understand the conceptual meaning of and be able to carry out the task correctly. The phrase’s level of formality is more appropriate to this age group.

‘Attractive’ and ‘visually appealing’ will be used to older students (KS3 +) to use vocabulary which is more specific to their communication skills.

The longest side of the pupil’s rectangle will be divided by the shortest side to give a ratio which is then recorded.

Students will not be able to see other participants’ rectangles.

Appendix 4- Questionnaire given to teachers

Type of School ……………………………………… **Position** ………………………………..

Year Groups Responsible for Teaching…………………………………..

Key: SD= Strongly Disagree **D**= Disagree **A**= Agree **SA**= Strongly Agree

	SD	D	A	SA
1. I enjoy teaching Geometry.	☐	☐	☐	☐
2. There is enough Geometry in the curriculum.	☐	☐	☐	☐
3. I teach Geometry more than other areas of Mathematics.	☐	☐	☐	☐
4 My pupils understand Geometry better than other areas of Mathematics.	☐	☐	☐	☐
5. Children enjoy Geometry more than other areas of Mathematics.	☐	☐	☐	☐
6. Geometry is quite a complex topic to teach.	☐	☐	☐	☐
7. Most of my Geometry lessons are practical.	☐	☐	☐	☐
8. Geometry is more important than other areas of Mathematics	☐	☐	☐	☐
9. I feel there should be more guidance and theory on how to teach Geometry.	☐	☐	☐	☐

10. Draw a rectangle in the box below:

Appendix 5- Ethical Checklist

Checklist of Ethical Principles for Research

Please tick

1. Accountability

	YES	NO
• Are there clear potential benefits?	☒	☐
• Are there procedures for obtaining written informed consent? If not, please explain:	☒	☐

Comments: It will inform my own practice and help many teachers and learners.

2. Confidentiality

	YES	NO
• Are there arrangements for ensuring anonymity? If not, please explain the nature of confidentiality protection of participants and institutions	☒	☐

Comments: The schools will be identified as being in the North East of England but will not be formally named (and this is given as relative anonymity) and all participants will be completely anonymous.

3. Anti-Discriminatory

	YES	NO
• Does the project demonstrate sensitivity to differences (e.g. political, religious, cultural)	☒	☐

Comments: Students are selected free of any bias such as gender, race and tasks are generic and non- offensive.

4. Reciprocal

	YES	NO
• Is the research mutual in its benefit and value to participants and researchers	☒	☐

Comments: Schools will benefit from expertise and results will improve researchers and many other practitioners' practice which will help many learners.

5. Empowering – Human Rights

	YES	NO
• Are participants given the freedom to express their needs including the right to refuse or withdraw participation?	☒	☐

Comments: All participation is voluntary- participants and schools can withdraw at any time/refuse to take part in study.

6. Honouring Professional Values – peer review

	YES	NO
• Does a professional code of conduct apply?	☒	☐
• If so, is it explicit in the research methodology?	☒	☐
• Has the project already been peer-reviewed? If so, by whom, and what was the outcome?	☒	☐

Comments: Anonymity is respected at all times- peers were impressed with original approach- most felt I had organised it well and found it an interesting idea.

7. Accessible

	YES	NO
• Is there a plan to make the results available and disseminate them in the public domain, particularly to stakeholders?	☒	☐

Comments: Schools will see results and could be published in the public domain subject to consent from **all** parties involved.

8. Challenging

	YES	NO
• Is the research seeking new knowledge and or insights?	☒	☐
• How is it doing this?		

Comments: Original approach- not much research done in this area. Compares all Key Stages (including A Level), which is quite rare to investigate, particularly the area of Geometry in it.

9. Appropriate use of Funding –

	YES	NO
• Could there be any conflict of interests?	☒	☐

Comments: There is no funding required to do this research and it is entirely independent- there is no vested interest involved.

10. Responsible

	YES	NO
• Is there a plan for the conduct of the research which ensures responsible behaviour?	☒	☐
• Are issues of Health and Safety considered	☒	☐
• Is the proposed analysis appropriate to the data?	☒	☐

Comments: All experiments take place in a safe and controlled environment- analysis is appropriate to data- a mixture of qualitative and quantitative.

Title of Project: SECC 6003 Researching Learning and Teaching- 'How do Children learn shape?'
Name of Researcher: Samuel James Curran
Name of Supervisor: Fiona Lawton
Date: 17/06/2013

Appendix 6- Written Information Sheet

Date:

INFORMATION SHEET

As part of my studies at the University of Cumbria, I have been asked to carry out a small study investigating how children learn shape.

I have approached you because I would like to ask your child to take part in this research project. The study will take about 10 minutes. I would be very grateful if you would allow your child to take part.

At every stage, your child's name will remain confidential. The data will be kept securely and will be used for academic purposes only.

If you have any queries about the study, please feel free to contact myself or my module tutor, **Fiona Lawton**, who can be contacted on **Fiona.lawton@cumbria.ac.uk** or by phone on 01524 384383.

Signed

Samuel James Curran

samcurran@live.co.uk

Appendix 6- Consent form

SECC6003

CONSENT FORM

Project title: SECC 6003 Research Study:

'Is the Van Hiele Model useful in determining how children learn Geometry?'

1. I have read and had explained to me by Sam Curran the Information Sheet relating to this project.

2. I have had explained to me the purposes of the project and what will be required of my child/class, and any questions have been answered to my satisfaction. I agree to the arrangements for their participation as described in the Information Sheet.

3. I understand that this participation is entirely voluntary and that I have the right to withdraw from the project any time.

4. I have received a copy of this Consent Form and the accompanying Information Sheet.

Name:

Signed:

Date:

Appendix 7- Raw Data from Investigation

Task 1

Pupil	Gender	Key Stage	Guess	Length of Line	Difference
1	M	1	14 cm	10 cm	4 cm
2	F	1	13 cm	10 cm	3 cm
3	M	2	16 cm	10 cm	6 cm
4	F	2	13 cm	10 cm	3 cm
5	M	3	11 cm	10 cm	1 cm
6	F	3	7 cm	10 cm	3 cm
7	M	4	10 cm	10 cm	**0 cm**
8	F	4	6 cm	10 cm	4 cm
9	M	5	8 cm	10 cm	2 cm
10	F	5	10 cm	10 cm	**0 cm**

Task 2

Pupil	Gender	Key Stage	Guess	Angle	Difference
1	M	1	91˚	120˚	29˚
2	F	1	60˚	120˚	60˚
3	M	2	90˚	120˚	30˚
4	F	2	80˚	120˚	40˚
5	M	3	105˚	120˚	15˚
6	F	3	180˚	120˚	60˚
7	M	4	130˚	120˚	10˚
8	F	4	120˚	120˚	**0˚**
9	M	5	120˚	120˚	**0˚**
10	F	5	120˚	120˚	**0˚**

Task 3

Pupil	**Gender**	**Key Stage**	**Orientation**
1	M	1	0˚
2	F	1	0˚
3	M	2	0˚
4	F	2	0˚
5	M	3	0˚
6	F	3	0˚
7	M	4	0˚
8	F	4	0˚
9	M	5	0˚
10	F	5	0˚

Task 4

Pupil	G	KS	Right-Angle	Scalene	Equilateral	Isosceles	Notes
1	M	1	2	2	2	2	Drew 4 isosceles triangles with similar ratios (i.e. similar shapes)
2	F	1	2	2	2	2	Drew 4 scalene triangles, 2 of which were congruent
3	M	2	4	4	4	4	Drew all triangles without prompts, misspelt types of triangle.
4	F	2	4	4	4	4	Drew all triangles without prompts, misspelt types of triangle.
5	M	3	5	5	5	5	Drew triangles really well, even with equivalence signs.
6	F	3	5	3	5	3	Drew some triangles with non-prototypical orientation.
7	M	4	5	5	5	5	All correct triangles with equivalence signs.
8	F	4	5	5	5	5	All correct triangles with equivalence signs. Very neat
9	M	5	4	3	4	4	Needed support on scalene triangle.
10	F	5	2	2	2	2	Knew all triangles without prompts and resized one as it 'did not look right'. Possible consideration of the beauty of the shapes.

Key:

1- Intensive support needed to understand task
2- Able to draw some triangles with prompts
3- Able to draw all triangles correctly with some prompts
4- Drew all triangles correctly with no prompts
5- Used extra mathematical notation such as equivalence signs

Task 5

Pupil	**Gender**	**Key Stage**	**D_1**	**D_2**	**D_1/D_2**
1	M	1	5.5 cm	2.3 cm	2.39
2	F	1	5.5 cm	2 cm	2.75
3	M	2	11 cm	4.3 cm	2.56
4	F	2	14.7 cm	4.1 cm	3.59
5	M	3	10.1 cm	5.3 cm	1.91
6	F	3	10 cm	2 cm	5
7	M	4	10.2 cm	4 cm	2.55
8	F	4	10 cm	4.2 cm	2.38
9	M	5	5 cm	4 cm	1.25
10	F	5	12 cm	8 cm	1.5

Key:
D_1= Longest Side of Rectangle

D_2= Shortest Side of Rectangle

Raw Data of Teacher's rectangles drawn in Questionnaire

Type of school taught at	D_1	D_2	D_1/D_2
Primary School	6.7	4.2	1.60
Primary School	13	3.5	3.71
Primary School	18	11	1.64
Primary School	4.1	1.4	2.93
Primary School	7.4	3.6	2.06
Primary School	5.7	3.6	1.58
Primary School	7.5	2.9	2.59
Primary School	3.1	0.9	3.44
Primary School	10.5	3.1	3.39
Primary School	5.9	2.1	2.81
Primary School	6.1	3.3	1.85
Primary School	5.5	2.9	1.90
Primary School	5.6	2.6	2.15
Primary School	3.7	2.4	1.54
Primary School	3.1	1	3.1
Primary School	1.6	1	1.6
Primary School	3.6	2.1	1.71
Primary School	3.9	0.3	13
Primary School	2.8	1	2.8
Primary School	18	11.5	1.57
Primary School	2.2	1.2	1.83
Primary School	4	1.6	2.5
Primary School	1.3	1.1	1.18
Primary School	6	2	3
Primary School	3	1.4	2.14
Primary School	1.2	1	1.2
Secondary school	5.5	3.3	1.67
Secondary school	4.1	2.1	1.95
Secondary school	10.3	5.1	2.02
Secondary school	7.1	1.9	3.74
Secondary School	3.2	3.2	1
Secondary School	6.1	4	1.53
Sixth Form	8.8	3.2	2.75
Sixth Form	8.2	3.5	2.34

Sixth Form	5.6	3.4	1.65
Sixth Form	6	2	3
Sixth Form	4	1.1	3.63

Key:

D_1= Longest Side of Rectangle

D_2= Shortest Side of Rectangle

Appendix 8 - Golden Ratio SPSS Data

Note: PRATIO = Longest side of students' rectangle/ Shortest side of their rectangle

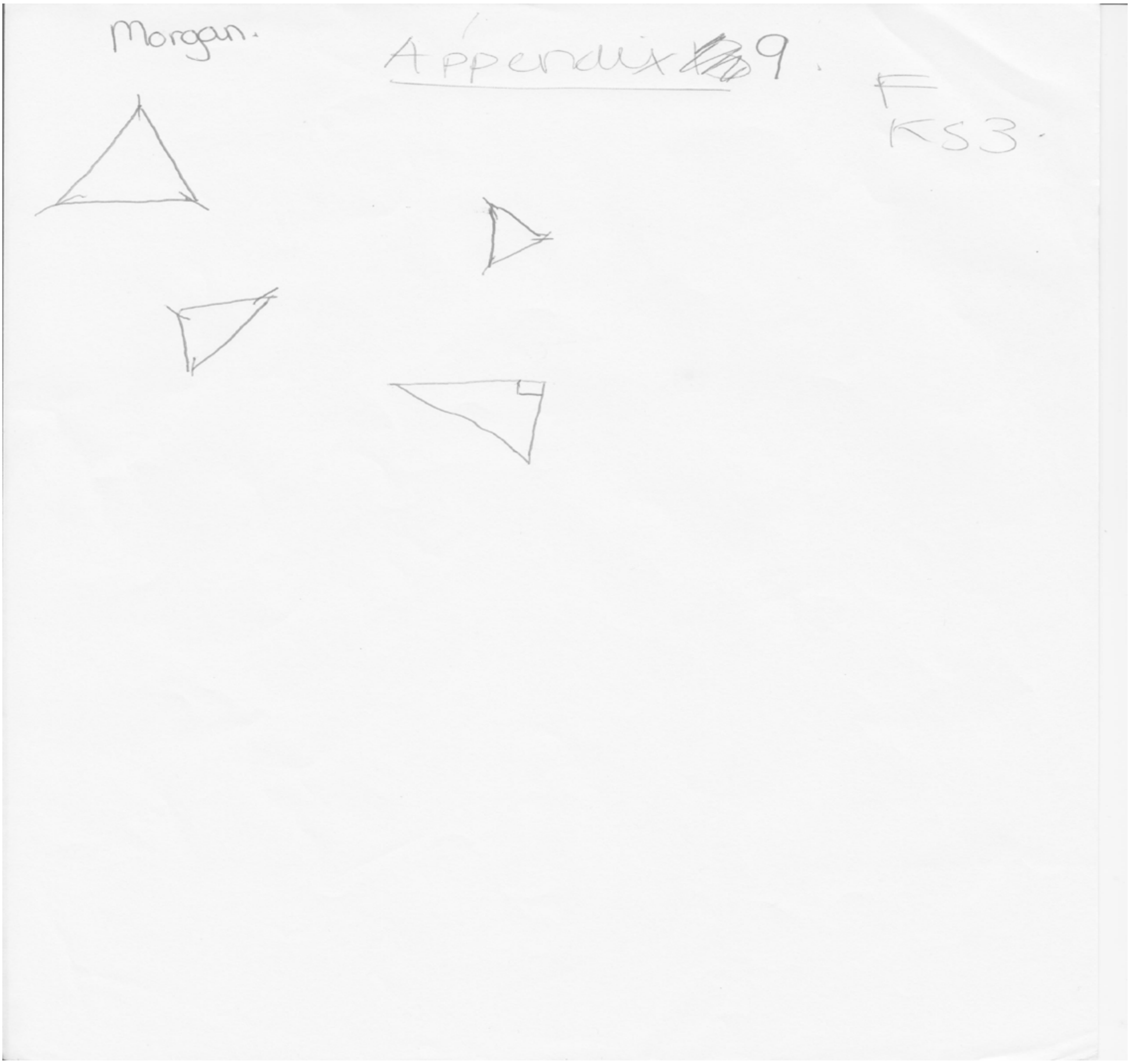
	PRATIO	var
1	2.39	
2	2.75	
3	2.56	
4	3.59	
5	1.91	
6	5.00	
7	2.55	
8	2.38	
9	1.25	
10	1.50	

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
VAR00001	10	2.5880	1.07323	.33938

One-Sample Test

	Test Value = 1.618					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
VAR00001	2.858	9	.019	.97000	.2023	1.7377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Appendix 9.
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KS3.

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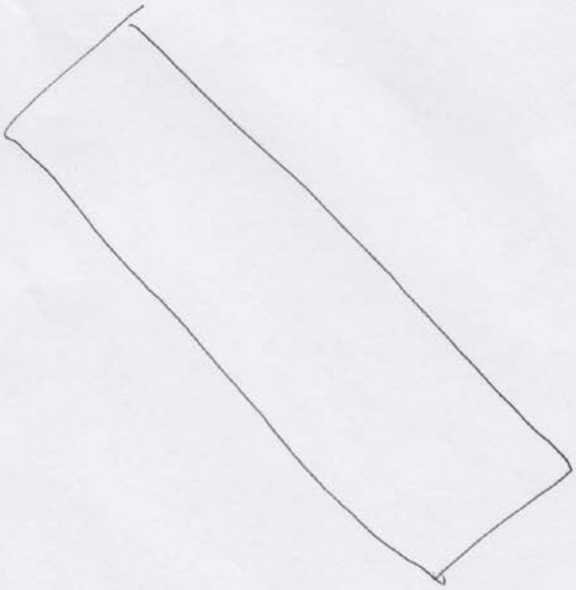

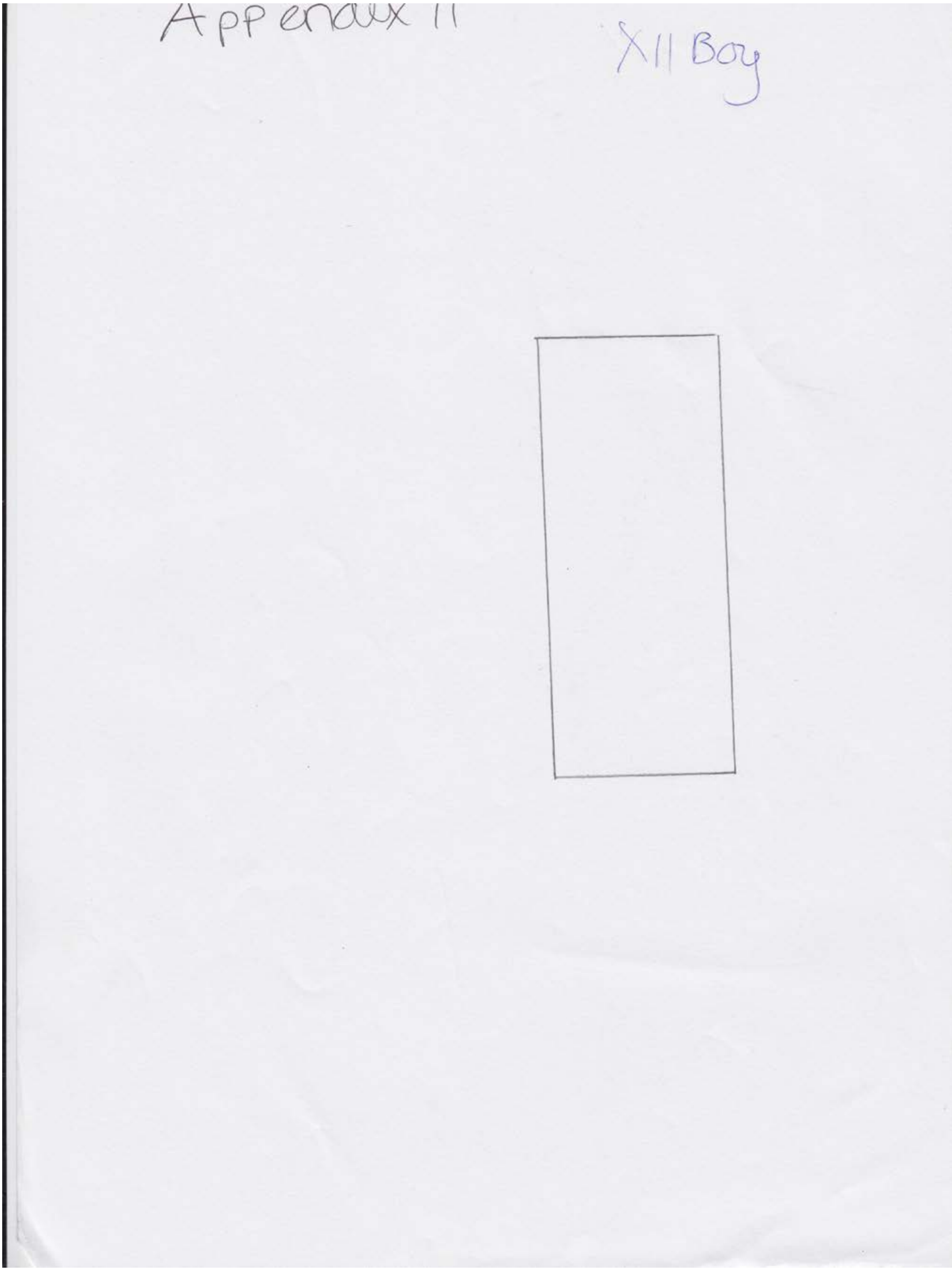
Appendix 11
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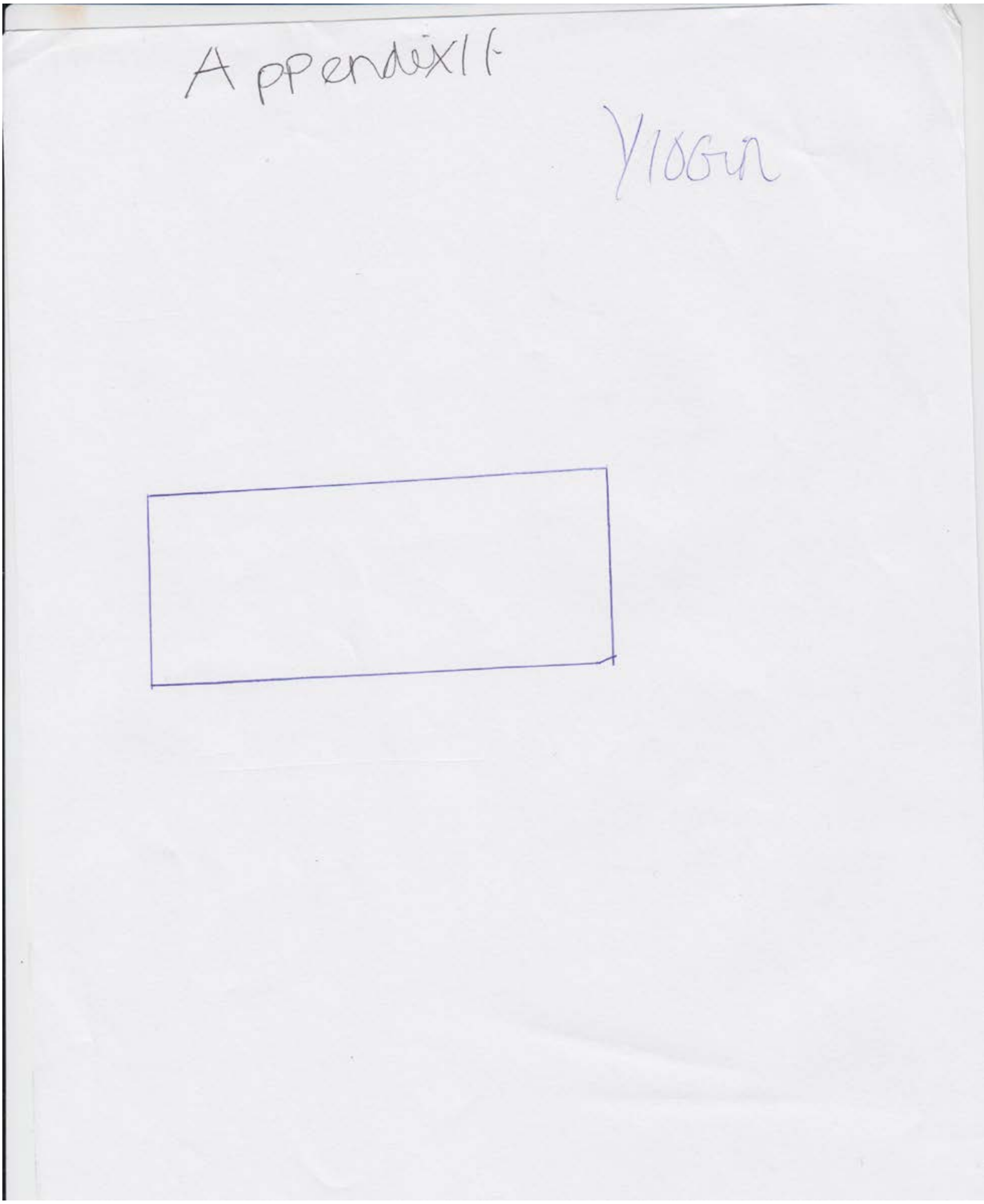
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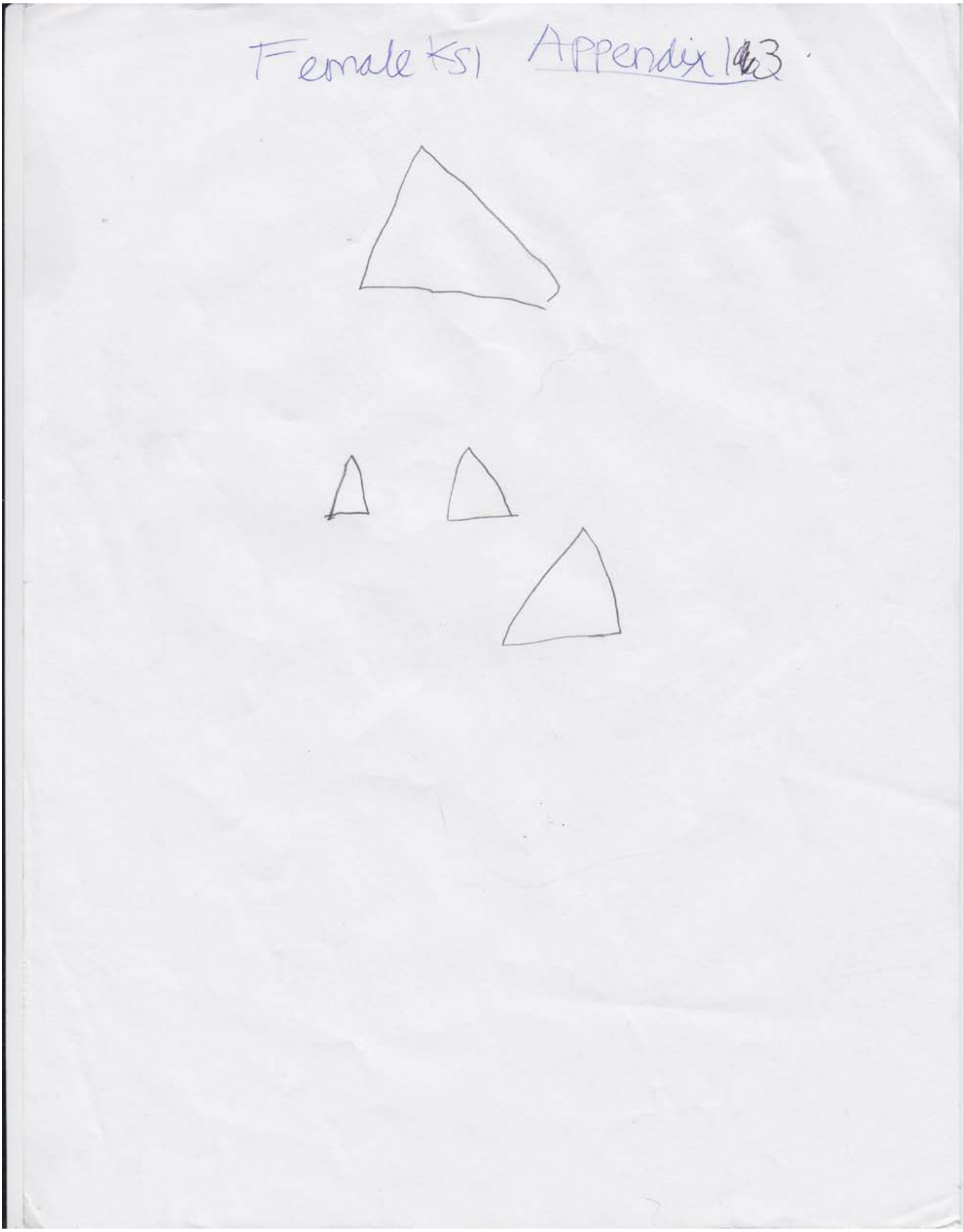
Appendix 12.

MKS1

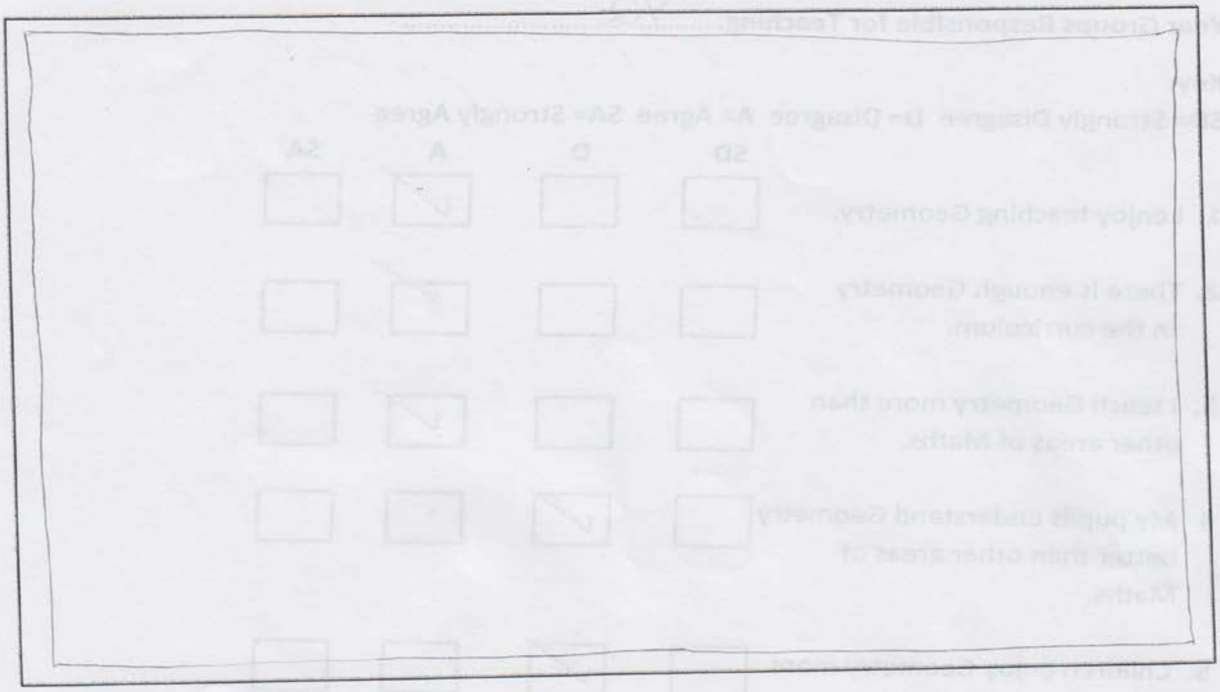

Female KS1 Appendix 13

Appendix 14.

10. **Draw a rectangle in the box below:**

Appendix 14.

10. **Draw a rectangle in the box below:**

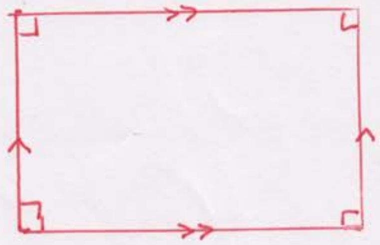


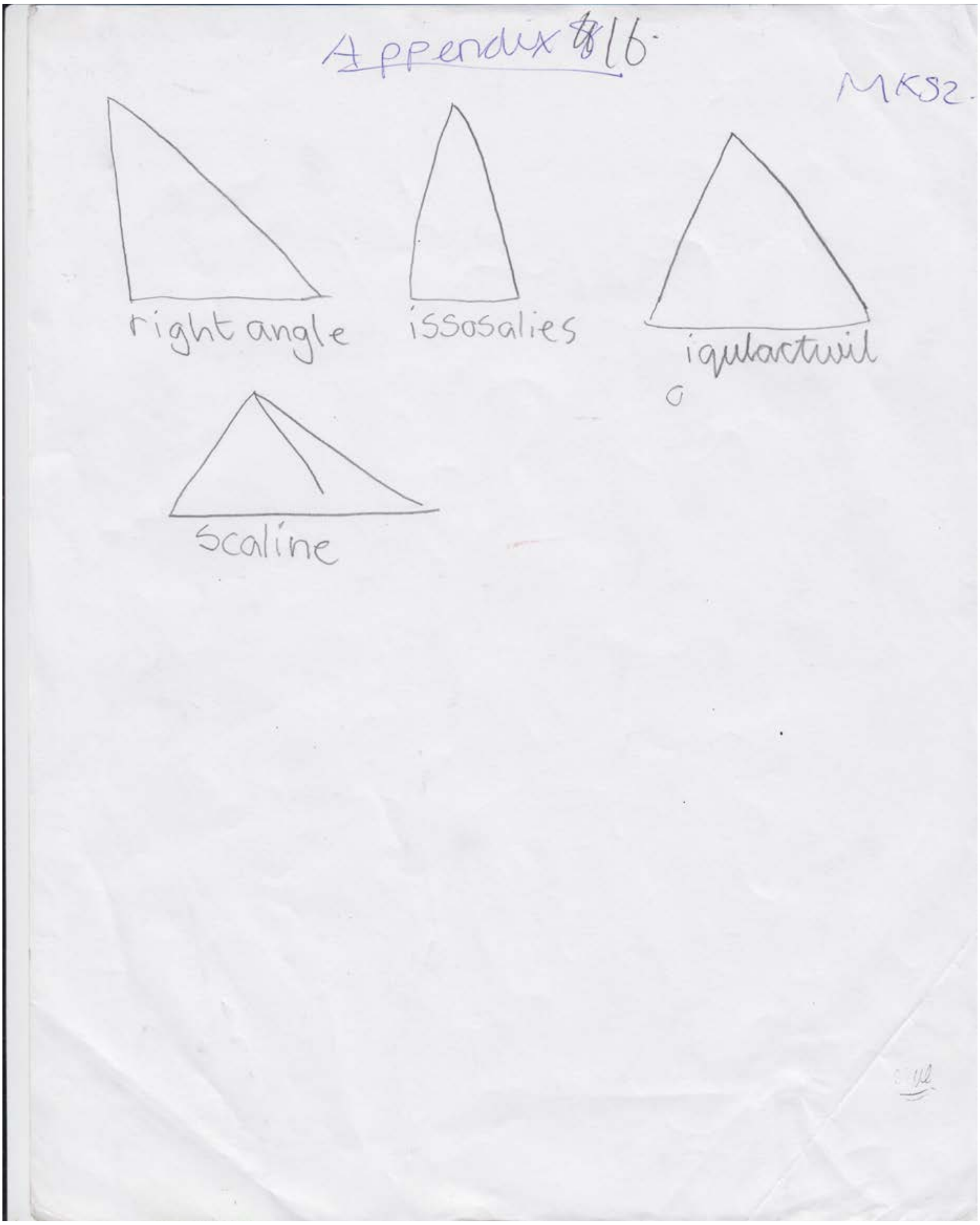
Appendix 5

10. Draw a rectangle in the box below:

Appendix 15

10. Draw a rectangle in the box below:




Appendix 16
MKS2.
right angle
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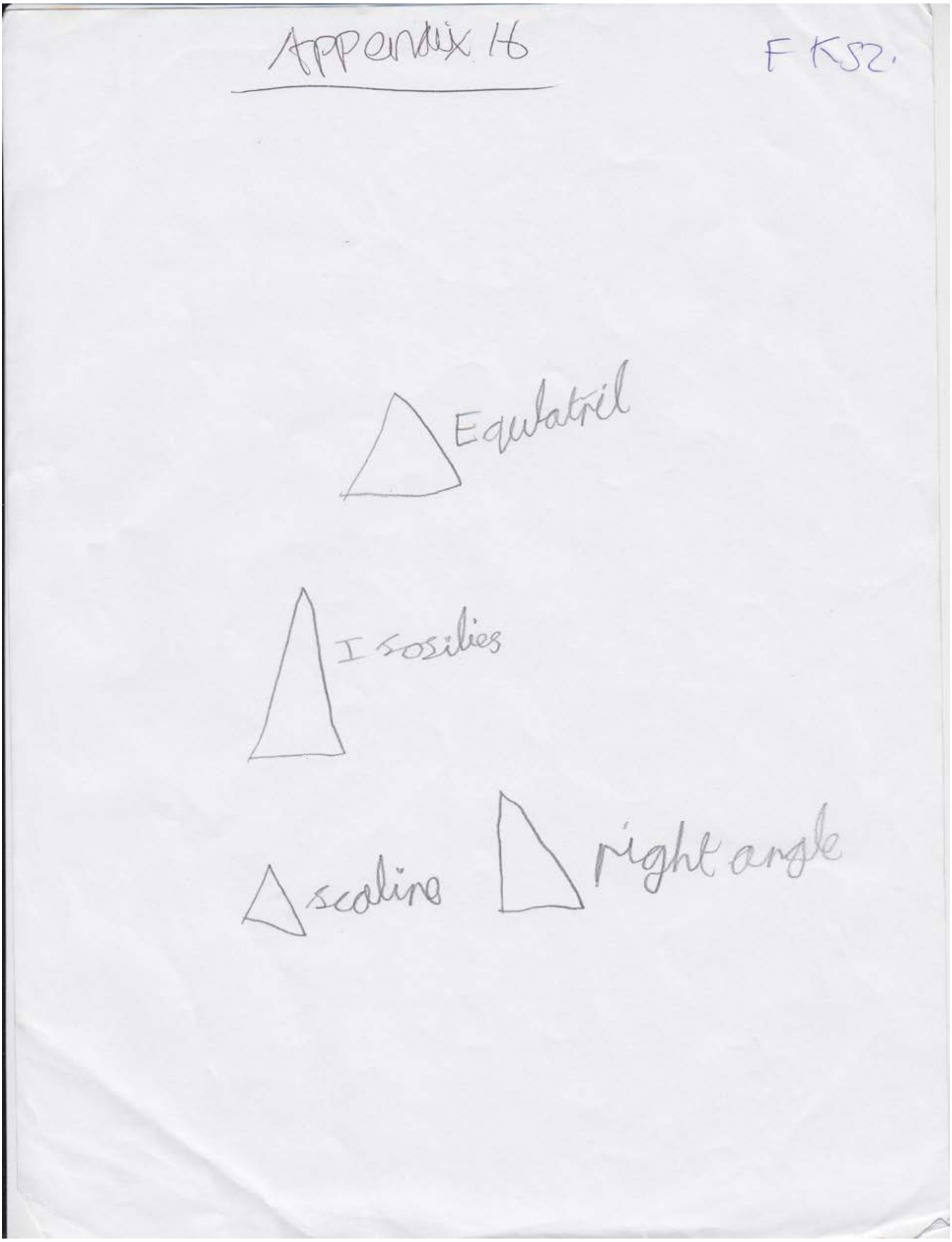
Appendix 16
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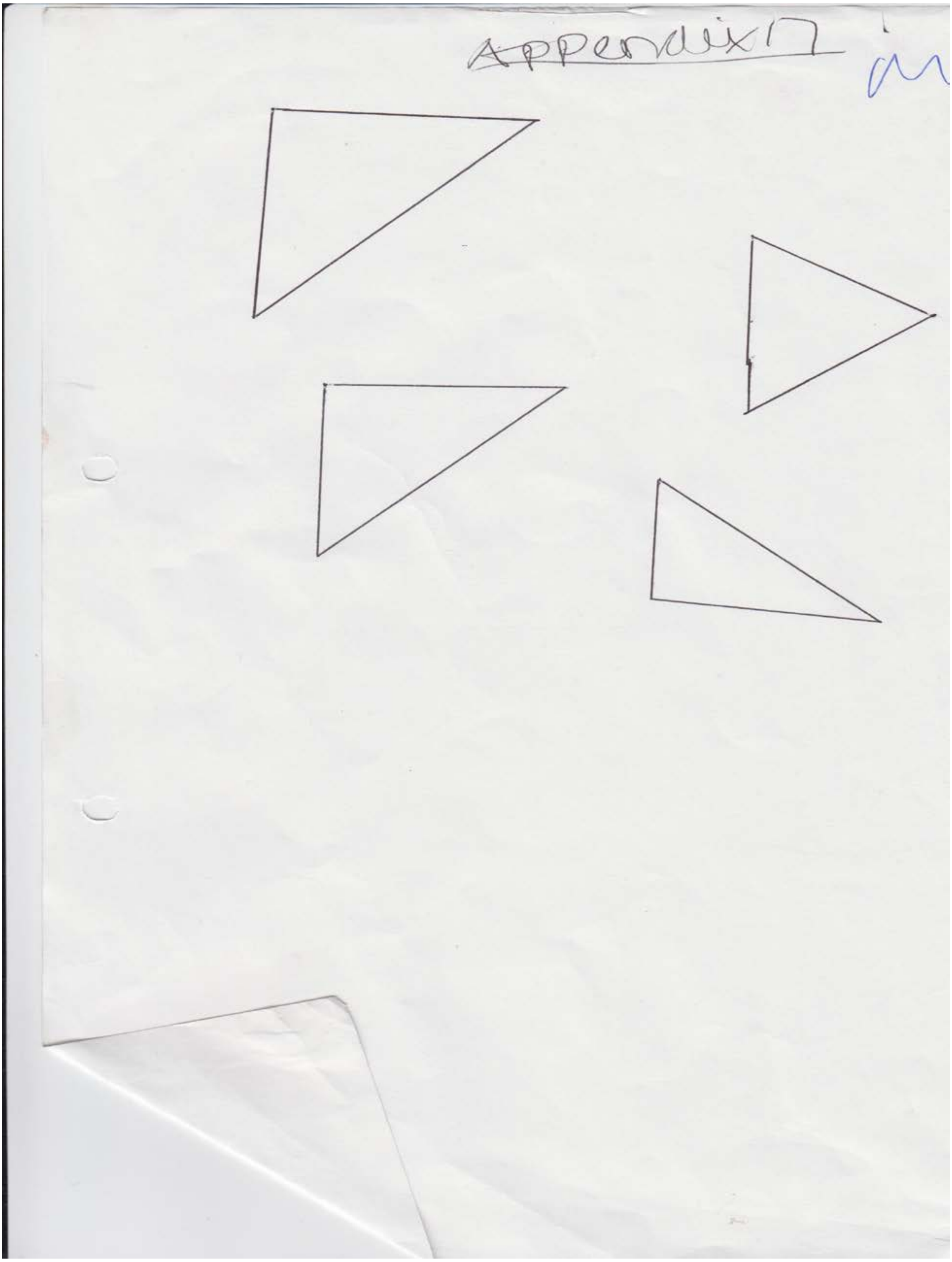
Appendix 17
FKS5


Appendix 17