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Primary Teacher Education

An investigation into what extent manipulatives influence the teaching of mastery mathematics based in a key stage two primary setting.

by

Stephen Mundy

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Abstract

This research project explores the effectiveness and the influence manipulatives have within the teaching and delivery of mastery in mathematics in Key Stage two in a Primary school. The research considered what teaching strategies are used within the practice of mastery in mathematics; additionally, it examines the benefits of manipulatives in the teaching of mastery mathematics and how this overcomes the learning barriers. Furthermore, how manipulatives impact individual's attitudes towards mastery mathematics in Key Stage two.

The research has examined previous literature to gain an awareness of the recent refocus within mastery mathematics. Additionally, this project identifies the provision of the manipulatives used within a primary school. The research tools which were used to collate the data includes semi-structured interviews; observations; questionnaires and focus group interviews. The close analysis of the data identified several teaching strategies which were used in conjunction with one another to be the most effective practice of mastery mathematics. It appeared that teachers believed manipulatives provided individuals with a method of visualising mathematical concepts while increasing their self-esteem, confidence and creativity.

From the conclusions of the findings, recommendations were formulated for the participating school; mastery mathematics programme and the governing board on how best to develop the mastery of mathematics in Primary schools. Finally, this research projects builds upon work outlined in the project proposal (Mundy, 2018).

Contents

Research Issue

The research issue considered the impact manipulatives have on the delivery and teaching of mastery across Key Stage Two. 'Mastery' is outlined as acquiring a deep, secure understanding of mathematical concepts that when adapted enable pupils to progress towards advanced concepts (NCETM, 2016). 'Manipulatives' are objects which can be physically handled with precision to gain an understanding of a mathematical concept (Watson, Jones and Pratt, 2013).

Justification

This research project has contributed competence towards achieving Teachers' Standards 2d and 3b: "demonstrate knowledge and understanding of how pupils learn and how this impacts on teaching" (DfE, 2013, p.1).

The personal relevance of this research is that it has provided an opportunity to impart the underpinning knowledge of mathematics to the younger generation, this corresponds with a personal affection towards the subject and the importance and ability to develop as a self-practitioner to teach mathematics in a real-life context through a 'hands-on' approach using manipulatives.

Another personal relevance is the ability to identify key strategies which can be used in teaching mastery mathematics to help pupils clearly understand the method and reasoning behind each mathematical concept (Clausen-May 2013).

The National Curriculum encourages mastery in mathematics as pupils should be challenged with sophisticated word problems to consolidate their understanding. Understanding in mathematics is identified as either knowing a process and the reasons behind the process (relational) or by knowing the process without the reasoning (instrumental) (Skemp, 2006). Word problems develop a fluency between the concrete representation and the mathematical skills required to gain mastery of a mathematical concept (DfE, 2013).

A complementary project which supports the National Curriculum's perspective is the Education Endowment Foundation, who concluded the impact of schools using the mastery approach on average, make two months' additional progress over schools who did not adopt this approach

(Vignoles, Jerrim and Cowan, 2015). Both documents demonstrate the importance mastery mathematics has on supporting pupils to make significant progress and acceleration.

Therefore, it is necessary teachers understand the impact manipulatives have on teaching mastery mathematics, this project is constructed to examine the approaches used.

In 2015, the PISA results identified that pupils from high performing countries in mathematics (Japan, Singapore, and Korea) could not just reproduce mathematical knowledge but apply it to various new situations (OECD, 2015). In Korea, research has established mastery learning in mathematics not only impacts the achievement of pupils but also promotes positive attitudes towards mathematics (Yildiran and Aydin, 2005).

Singapore Maths (an initiative adopted in UK schools based around the Singapore curriculum), is an effective scheme which allows pupils to explore confidently and freely with manipulatives to gain understanding of mathematical knowledge and apply to problem solving (Kho, Yeo, and Lim, 2009). Significant investments have been made internationally with mastery mathematics research, however, more investment nationally is required to establish the importance of manipulatives within mastery mathematics.

Research Questions

To what extent does manipulatives influence the teaching of mastery in mathematics in primary schools?

Research questions are formulated to investigate and explore a specific topic or area within a field of interest by a researcher, the questions will sub-divide the objective of the study and support to find out the answers relating to the main topic question (Holliday, 2007).

1. **What teaching strategies are implemented in the delivery of mastery in maths in KS2?**

2. **What are the benefits of manipulatives in teaching mastery mathematics in KS2 which overcome the barriers for learning?**

3. **How do manipulatives impact the pupil’s attitudes towards the mastery of mathematics in KS2?**

Literature Review

Introduction

This literature review investigates what extent manipulatives impact on teaching mastery mathematics. I have selected this area as the delivery of mathematics is of great importance and a core subject in education (Lundin, 2012). There has been a recent refocus on mastery in mathematics, this is important to identify how children visualise mathematics using manipulatives. The findings from this literature review will develop my own teaching practice within mathematics.

Mastery

The Department of Education (2015) defines mastery as deep and secure learning of a mathematical concept. Complementary of this, the National Centre for Excellence in the Teaching of Mathematics (2015) categorise the term mastery into four elements: mastery approach; mastery curriculum; teaching for mastery and achieving mastery of areas in mathematics. Furthermore, the National Curriculum (2013) ties in the concept of mastery as pupils move through the programme of mathematics at the same rate and gain deeper understanding of mathematical concepts by being able to solve complex problems and reason mathematically with fluency.

The Department of Education (2016) acknowledges the lack of development in the UK's performance between 2012 and 2015 PISA results for collaborative problem solving. Countries who performed the best in the PISA 2015 collaborative problem-solving tests were Singapore and Japan, with Finland being the best performing European country (OECD, 2015). This has been accredited by the high amount of problem-solving opportunities provided in each country (Fan and Zhu, 2007; Fernandez and Yoshida, 2012 and Andrews *et al,* 2014).

However, Jerrim and Wyness (2016) criticise the reliability of the PISA results due to the variation of results between computerised and paper-based tests. In comparison, Hopfenbeck (2016) expresses that PISA does not take into consideration the population of a country when choosing a selective sample size. To conclude, the 2015 Trends in Maths and Science Study results highlight an improvement in England's year five collaborative problem-solving mathematics performance between 1995 – 2015 (DfE, 2016). Therefore, this brings into question the validity of the PISA results.

Guskey (2007) analysed Bloom's mastery learning theory from the 1960's and concluded that mastery provides an opportunity for all ability individuals to achieve and reduces the gap of achievement between the lower and higher ability individuals. In contrast, Ollerton (2009) claims there is a fixed theory of ability in mathematics, that you are either good at mathematics or not, which can negatively impact an individual's expectation of achievement. However, a major drawback of Ollerton's analysis is it does not provide evidence of the negative impact.

In resolution to this, Dweck (2015) persuasively suggests individuals who develop their mathematical abilities through dedication (growth mindset) will achieve more than individuals with a fixed mindset as they identify misconceptions and failures as an indication to develop their mathematical potential. The benefits of growth mindset in mastery mathematics have also been acknowledged by other researchers (see Boaler, 2015 and Drury, 2018).

Elements of Mastery

The National Centre for Excellence in Teaching Mathematics (2014) revealed intelligent practice, variation, limited differentiation and connections of representations as some of the key elements in mastery.

Ma (2010) and NCETM (2015) define intelligent practice in teaching mastery as the precise design of mathematical activites, resources and questions, which promotes creative thinking and avoids repetitive learning. Opposing this, Carruthers and Worthington's (2006) study on teachers' perspective on creative mathematics concluded that 80% of respondents could not provide an example of an individual's creativity in mathematics.

Despite this, a criticism of Carruthers and Worthington's (2006) conclusion is that it lacks reliability as there has been no attempt to recomplete this approach within a primary school environment. In conclusion, the National Association of Mathematics Advisers (2015) identify that variation is used in mastery mathematics as individuals use a concept or procedure in a range of contexts, this promotes creative thinking, procedural fluency and conceptual understanding (Ryan and Williams, 2007 and NCETM, 2016).

Stripp (2014) identifies an incompatibility between mastery mathematics and differentiation of mathematical content for ability grouping. A more comprehensive explanation is provided by Johnston-Wilder *et al,* (2005) and NCETM, (2014) who imply differentiation in mastery mathematics as immediate interventions to address misconceptions and skilful questioning to enrich an individual's knowledge. This method of differentiation is integrated into the National Curriculum (2013) as it accommodates the needs of all pupils providing them with the best opportunity to succeed in mathematics.

Furthermore, the National Textbook Project (NCETM, 2015) and Drury (2018) conclude differentiation in mastery mathematics provides access for all individuals to attempt the same activity and by doing so, focuses on content depth which can benefit all pupils through the emphasis on accuracy and mathematical connections.

Montague-Smith and Price (2012) present Bruner's three staged model of representations, – enactive (concrete i.e. manipulatives), iconic (pictorial i.e. diagrams) and symbolic (abstract i.e. language) representations, as a model which coincides with mastery. However, Montague-Smith and Price (2012) fail to acknowledge the equilibrium required between the use of each representation. Furthermore, Coles and Copeland (2002) suggests without this connection between each representation, the learners' opportunity for mastery are limited.

In resolution of this, Haylock and Cockburn (2013) establish that representations need to be worked with extensively in a variety of contexts to gain a deeper understanding between the concrete, pictorial and abstract stages. More specifically, Carroll *et al* (2007) highlight using a variety of concrete apparatus in a real-world context allows individuals to visualise abstract mathematical concepts, this visualisation also increases an individual's self-esteem in mathematics. The benefit of using mathematical resources in a real-world context has also been identified by other researchers (Drews and Hansen, 2007; Clausen-May, 2013 and Haylock and Manning, 2014).

Concrete Resources

Brown and Liebling (2014) define concrete resources as a physical object which can be used to visually represent a mathematical concept. Glazzard (2016) supports Brown and Liebling's (2014) definition and goes further by recognising Piaget's concrete operational stage, as individuals interact with concrete resources and simultaneously consider multiple dimensions of the resource to create a schema of mathematical concepts.

Bottle (2006) provides a critique of the use of concrete resources, by stating the teacher cannot just provide the resources and believe that effective learning will take place. This questions the role of the teacher when using concrete resources in mastery mathematics. Schoenfeld (2014) offers the most acknowledgeable solution by clarifying that individuals will have a personalised approach in exploring concrete resources, but teachers must facilitate the learning through scaffolding techniques to enhance the individual's learning. The scaffolding techniques used would be modelling, high-level questioning and oral feedback (Anghileri, 2006 and Briggs *et al,* 2014).

Moyer (2001) identified that some teachers do not provide concrete resources in mathematics, even if they are beneficial to the individual's mathematical learning as the concrete resources are deemed as childish. Limitations of this viewpoint are that it fails to specify the age of the individuals and explain how a concrete resources are deemed as childish. Furthermore, Drews and Hansen (2007) analysed

Moyer's (2001) conclusion and established that the underlying issue was the teachers had a lack of understanding on how to use concrete resources to represent the mathematical concepts.

Harris and Taylor (2013) directly oppose Moyer's (2001) viewpoint by suggesting that older individuals would use concrete resources more in mathematics if it involves problem-solving activites. This claim is supported by Van Den Heuvel-Panhuizen and Buys (2001) as in the Netherlands, older individuals start the lesson with a problem and use concrete resources to initiate logical thinking to resolve the problem. Additionally, in Singapore Maths, older individuals must make purposeful choices about which concrete resources they use to tackle problems as this provides them with an ownership of the curriculum (Ash and Boyd, 2018). In conclusion, Ofsted (2017) specify that inspectors focus on observing how concrete resources are used in lessons and approaches applied to enable children to master mathematics.

Manipulatives

Manipulatives are objects which individuals can perceive an understanding of mathematical concepts through an appropriate hands-on experience of precise movement and handling (Griffiths, Back and Gifford, 2016). The use of manipulatives originates from Maria Montessori's approach towards mathematics as carefully structured materials were manipulated to provide individuals with a sense of mathematical understanding (Furner and Worrell, 2017).

Thompson (2010) expresses that individuals need to be provided with plenty of opportunities to investigate and explore freely with concrete manipulatives. However, the generalisability of Thompson's viewpoint is problematic as it fails to address benefits of exploring manipulatives. Contradictory of Thompson (2010), Kelly (2006) advises teachers need to continually facilitate individuals with manipulatives to ensure they are being used for the correct purpose and not as a distraction mechanism from the mathematical learning.

Clements (2000) agrees with Kelly's (2006) viewpoint and establishes the importance of teacher's continual development towards using manipulatives as attitudes and achievement are improved in mathematics when individuals have clear, precise instruction and exploration of concrete manipulatives. Ofsted (2012) persuasively highlight that effective use of manipulatives in conjunction with a range of teaching strategies in mastery mathematics can create a stimulating learning environment which aids individuals conceptual understanding.

The NCETM (2017) indicate representation and structure as one of five elements in teaching for mastery, as using manipulatives provide practical experience to visualise connections between concrete and abstract concepts. This visualisation using manipulatives supports children's development of mental fluency and conceptual understanding (DfE, 2013).

Furthermore, Brown and Liebling (2014) provide a useful insight by observing that individuals developed greater levels of resilience and confidence when using manipulatives as it provided new and interesting representations of mathematical concepts. Higher levels of resilience when using manipulatives has been supported by other researchers (Pham, 2015 and Pound and Lee, 2015). However, one major limitation of manipulatives is the availability in a mastery mathematics classroom. Little (2009) persuasively concludes if the manipulatives are unavailable then finding an equivalent to correctly reflect the mathematical concept as a concrete representation can be challenging.

Conclusion

In conclusion, there are critiques about the use of manipulatives in mastery mathematics, however, the literature has identified a consensus that manipulatives strongly correlate and impacts on the development of individuals conceptual knowledge and understanding in mastery mathematics.

Methodology

The type of research which has been conducted in this research project is a case study. A case study is an in-depth investigation which describes and interprets the development of a singular individual, group or system (Yin, 2013). The research instruments which have been used are questionnaires; semi-structured interviews; focus group interviews and semi-structured observations. All the research instruments produce qualitative data with questionnaires providing an aspect of quantitative data.

Qualitative data is information which is expressed through language description which uncovers understanding behind and individual's perspective (Silverman, 2015). However, Quantitative data is the collection of information which is expressed numerically (Bryman, 2016). As all the research instruments produce qualitative data this links to the interpretivist view of research. Interpretivism in qualitative research is the belief in multiple perspectives of reality, these multiple realities are then interpreted to obtain the understanding of an individuals' experiences (Ivankova and Plano Clark, 2016).

The research instruments were piloted before the research was conducted. Piloting is an experimental trial on the proposed research instruments using a sample of participants to highlight any potential problems (Kelley *et al*., 2003). Complementary of this, Knight (2001) establishes the effectiveness of piloting as it determines the best design of presenting the research instruments. This viewpoint is evidenced as the piloted questionnaire was adjusted to ensure that it accurately addressed the research questions.

Questionnaires are sets of questions which are designed to provide a written response from participants, these responses range from ticking a box to writing sentences (Lambert, 2012). Additionally, Denscombe (2014) recognises the purpose of questionnaires as they are designed to collate information for analysis.

The questionnaires were used to answer the first and second research questions. The sample for the questionnaires were twelve key stage two teachers from the chosen school, this consisted of three teachers from each year in key stage two. The questionnaires were handed out personally and completed questionnaires were collected face to face two weeks later.

One of the main strengths of the questionnaire process was how selective the questions were constructed, this enabled specific information to be collated from the participants. This is identified by Basit (2010) who stated that if a questionnaire is formulated efficiently then it can offer an accessible way to gather large amounts of relevant information with no time constraints.

In conjunction with Basit, Walliman (2011) agrees that questionnaires are an economic research tool and expresses the importance of delivering and collecting questionnaires personally as this allows for a higher response rate. Walliman's (2011) viewpoint highlights a main advantage of the questionnaire process as eight out of the twelve participants responded, the amount of responses could be attributed to the personal collection and delivery of questionnaires.

One of the unexpected issues which arose during the questionnaire process was some of the participants misinterpreted the meaning of the questions. White (2008) expresses how the inflexibility in the structuring of questions once handed out is a concern, as individuals can perceive the questions differently. To overcome this issue, there were a selected sample of participants chosen from the questionnaire responses to complete a semi-structured interview, these participants could then elaborate on any misinterpreted answers during the interview process. In conclusion, Blaxter, Hughes and Tight (2010) suggest using more than one research tool will improve the validity of the results allowing further exploration of a detailed perspective of any issues raised.

Semi-structured interviews are structured interactions between an interviewer and interviewee which has a careful questioning and response process which can be elaborated on if required to obtain knowledgeable responses (Kvale, 2008).

The semi-structured interviews were used to answer the first and second research questions. The participants for the semi-structured interviews were chosen to elaborate on the answers they provided within the questionnaire. The selection of the sample took into consideration a consensus of viewpoints on mastery by aiming for a participant from each year group in key stage two. The four semi-structured interviews were conducted face to face with the content being recorded for later transcription.

The main advantage of the semi-structured interviews is that it enabled the interviewer to selectively move through the questions, accessing the most appropriate information from each answer and providing freedom for further elaboration if necessary. This advantage is evidenced by Grieg, Taylor and Mackay (2007) and Macintyre (2012) who conclude the structure of semi-structured interviews allows interviewee's ideas to develop providing a large scope of information for the interviewer to analyse. Furthermore, Macqueen, McLellan and Neidig (2003) identify with the interviews being recorded and transcripted allows the interviewer to gain accurate information by not diverging from the interview agenda.

The issue which occurred was that the semi-structured interview and the transcription process was time consuming. In addition, due to the nature of the research, there was a bias towards the perceptions of the questions. Irvine, Drew, and Sainsbury (2013) define interviewer bias as when the interviewer has preconceptual prejudice questions and demonstrates this through the phrasing or tone of the questions.

To resolve the time constraints of the transcription process, the recordings were listened to continuously, with synopsis of each of the questions transcripted. In negotiating the interviewer bias, when verbally delivering the questions there was a consistency across the four interviews. Consistency of delivering questions to all participants in addressing interviewer bias has been acknowledged by a range of authors (Harrell and Bradley, 2009; Grix, 2010 and Rowley, 2012).

Focus group interviews are defined as a small pre-determined group that discuss and debate the research issues presented and the results are determined by the individual's consensus (Holliday, 2007).

Focus group interviews were used to focus on the third research question. Prior to this, a letter of consent was sent out to year five participants to gain permission to partake in the focus group interviews. Eight participants were then selected from the individuals who had given consent. Each participant then answered the focus group questions with the consensus being used as the answer.

The positives of the focus group interviews were that it provided a variety of participants' perceptions into mastery mathematics due to the sample size. Blaxter, Hughes and Tight (2010) acknowledges that diverse and in-depth views are generated if the individuals feel comfortable during the interview process. In addition, Eley *et al,* (2017) provide more clarity to Blaxter, Hughes and Tight's (2010) viewpoint by identifying that participants feel more secure if they are in groups and therefore more open in presenting their thoughts and attitudes towards the interview agenda.

However, a problem which occurred with the focus group interviews was that initially all the individual's responses were similar, therefore reducing the quality and variance of data (Hopkins, 2007). Complementary of this, Arthur, Coe and Waring (2012) conclude that social pressures in a focus group environment can affect the verbal and non-verbal responses which participants provide, this then results in a lack of deviation in the results. This clearly reduces the validity of the results collated from the focus group interviews, however, it is compensated by the triangulation of other research instruments.

Observations are where the researcher uses all their senses to record an area of interest, the semi structure to an observation means that there is a specific theme or idea which would be observed in the event due to the relevance it has in the research (Lambert, 2012). The semi-structured observation took place within a mastery mathematics lesson in year five. The observation took place as a non-participant and any events which correlated with the research questions during the lesson was summarised at five-minute intervals and transcripted onto the observation document.

The positive of the semi-structured observation was that due to the non-participant status in the observation, the sample was always observed, therefore ensuring that all events during the mastery lesson were recorded. In comparison, Mason (2002) highlights an advantage to observations, as you do not rely on a participant's ability to provide information. Furthermore, Corbin and Strauss (2008) illustrate that the benefit in a non-participant observation is that the observer is objective. This method of observation strengthens the validity of the results as by being objective, reduces the observer bias (Frankel and Wallen, 2008).

An unexpected issue which arose during the observations was that the sample wanted to interact with the observer during the lesson. Wisker (2009) establishes that if participants establish interactions with the observer then this will likely affect the participants behaviour. As a result, this change of behaviour can produce the data from the observation to be artificial (Bryman, 2016). To overcome this issue, the class teacher and teaching assistant provided the support to the participants, allowing a non-participation observation to take place. This ensured that the participants behaviour did not change during the observation.

Finally, the method of analysis which was used was a structured thematic approach. This was to look at the similarities and differences between each of the data sets and cross reference between each research tool to analyse the findings for each research question. Moore (2006) defines by using a structured analytical framework to evaluate your findings will allow a clearer conclusion of results and will be less time consuming.

In conclusion, all the research tools were originally deemed appropriate for this project, however upon reflection the focus group interviews did not provide a detailed response due to the social pressures in the group. In contrast, the other research tools were suited to the project due to the wide spectrum mastery covers and allowed individuals to express a detailed viewpoint about mastery.

Validity & Reliability

Validity is defined by the close measurement of what the research tool is set out to measure, providing a credible result from the data (Sapsford and Jupp, 2006). Greig, Taylor and Mackay (2007) identifies triangulation as a method to strengthen validity and defines triangulation as using two or more research tools within research. This relates in the research project, as all four research tools were used to achieve triangulation.

Seale (1999) and Knight (2002) diagnose a potential threat to validity about the structuring of questions and whether the questions reflect the required outcome of the investigation. However, Bell (2010) strongly negates this by asserting if piloting is used then a rigorous structure will be implemented. Before conducting the research, all four research tools were piloted, this provided a valuable insight into the research process of each instrument and ensured that it administered the most appropriate outcome. After the piloting process, the research instruments were edited to be more adaptable towards the research question.

Reliability is the accuracy of producing consistent results under the same conditions (Schostak, 2002). A threat to reliability is the trustworthiness of the information provided by the participants when using research tools which collect qualitative data (Morse *et al,* 2002), Contradictory to this, Bell (2010) identifies by assuring anonymity and confidentiality, participants will be more likely to provide honest responses which strengthens the reliability of the research. Therefore, when the research was conducted, the participants were assured of anonymity and confidentiality.

Finally, Denscombe (2014) invalidates using secondary data for analysis as if another researcher completes the investigation there would not be comparable findings. Therefore, in the research project, primary data will be personally collated from the participating school and used for analysis.

Ethics

Grix (2010) interprets research ethnicity as the conduction of the researcher to the participants on treatment of respect, professionalism, and privacy. Before the research was conducted, the structure of the research required approval from the University of Wolverhampton Ethical Consent Panel. Furthermore, permission was given by the research venue for the research to take place within their school.

Blaxter, Hughes and Tight (2010) decisively state that by keeping participants anonymous and their information confidential, this will maintain high standards of ethics. Therefore, to maintain high standards of ethics in the research, a consent form was sent out to participants prior to conducting the research. This required a signature in agreement to participate in the research and at any time throughout the research process the participants had the right to withdraw from the research.

In accordance with Walliman (2011), when collecting qualitative data from the interviews, all data independent to that participant will be presented back to them and in negotiations with the participant any uncomfortable information will be deleted.

Finally, Lambert (2012) examines data security as an ethical issue and clarifies about keeping any information under lock if paper based and to keep secure records of identities. Therefore, all paper-based documents were kept secure to maintain high standards of ethics.

Presentation & Analysis of data & discussion of findings

What teaching Strategies are implemented in the delivery of mastery in mathematics in KS2?

The research instrument used to answer this question is semi-structured observations, these findings were justified by teacher questionnaires and semi structured interviews.

The observation identified discussion partner talk; pupil demonstrations; use of pictorial representations and manipulatives as the teaching strategies used in the mastery mathematics lesson. In addition, there was a clearer focus in the observation on the use of manipulatives, as at thirty minutes, there was a misconception on multiplying fractions and the physical fraction blocks were used to address this.

The use of manipulatives in the school was identified in the questionnaire as 6 of the 8 respondents ‘agree,’ or ‘strongly agree,’ that the school has enough manipulatives available to use when teaching mastery mathematics.

The semi-structured interviews established collaborative learning with mixed ability groupings and problem-solving opportunities as other teaching strategies which are implemented in mastery mathematics. The questionnaire established that 7 out of 8 respondents ‘agree,’ that they have a knowledge of teaching strategies to use when employing a mastery approach.

In comparison, in the observations it was identified that the teaching strategies were used in conjunction such as partner talk used with manipulatives. This view is further evidenced by the participant in the first semi-structured interview as stated that concrete resources are used as a collaborative learning opportunity which develops into identifying abstract representation to use in problem solving.

However, In the observation, while the class was given manipulatives, the higher ability used pictorial fraction wall. Furthermore, in the fourth semi-structured interview, the participant stated when manipulatives were provided to the most able pupils, some individuals do not understand what to do with the manipulative.

Findings from the observation, questionnaire and semi-structured interviews have revealed wide variety of teaching strategies used in mastery mathematics and usually the teaching strategies are used in conjunction with one another, this finding confirms the literature of Ofsted (2012) who agree a variety of teaching strategies should be used in conjunction with manipulatives to create a stimulating learning environment. However, Little (2009) was challenged by the findings in the questionnaire as six teachers ‘agreed,’ or ‘strongly agreed,’ about the availability of manipulatives for the classroom.

The findings highlighted that the higher ability participants were not provided with manipulatives as often and when they did, they did not understand how to use them. This confirms the literature from Coles and Copeland (2002) that without connection between concrete to abstract representation it limits the learner’s opportunity for mastery.

In conclusion, the findings from the questionnaire, semi-structured interviews and observation contrasts Ma (2010) and NCETM (2015) viewpoints as there was no mention of the use of intelligent practice as a teaching strategy.

What are the benefits of manipulatives in teaching mastery mathematics in ks2 which overcome the barriers for learning?

This research question will be answered from the responses of the questionnaires, in addition the responses will be validated by the lesson observation and the semi-structured interviews.

100% of teachers 'agreed,' in the questionnaire that manipulatives are used to scaffold the pupils learning in mastery mathematics. This is expanded further in the four semi-structured interviews as it was identified that a benefit of manipulatives is that it allows pupils to physically see the mathematical concepts. Furthermore, the mastery lesson observation highlighted that manipulatives were also used to consolidate pupils' misconceptions in gap tasks.

In regards of the barriers for learning with manipulatives, the third semi-structured interview states that distractions can occur with manipulatives as pupils are unfamiliar with using them for the correct mathematical purpose. However, the results from the questionnaire show that 7 out of 8 respondents either 'agree,' or 'strongly agree,' that manipulatives positively contributed towards pupils' progression.

The fourth semi-structured interview details to overcome the barriers for learning, manipulatives need to be carefully chosen for each mathematical concept. This is validated by the results collated from the questionnaires expressed in table one, as it expresses out of the eight respondents, counters, base ten Diens and dice were the most used manipulative but on average the base ten Diens and numicon were the most effective manipulatives.

Table One

Resource	**Number of Teachers that use resource**	**Average Rating Scale [1 = Most Effective, 5 = Least Effective]**
Cubes	7/8	2.9
Counters	8/8	2.6
Base Ten Diens	8/8	2.0
Numicon	4/8	2.0
Place Value Slider/ Arrow Cards	7/8	2.6
Number Rods	2/8	2.5
Dice	8/8	3.8

One of the main benefits of manipulatives was outlined in the interviews as the teacher's consensus was that manipulatives provide individuals with a concrete representation of a mathematical concept. This benefit was verified further in the lesson observation as pupils manipulated the fraction wall (concrete representation) and demonstrated the abstract concept of multiplying fractions by representing the answer in a bar method (pictorial representation).

The questionnaire responses; lesson observation and the semi-structured interviews have identified that manipulatives can be used to understand the concrete, pictorial and abstract representations of a mathematical concept.

This finding approves the literature of Montague-Smith and Price (2012) who presented Bruner's three staged model of representations. Complementary of this, the finding clearly confirms the NCETM (2017) theory that representation and structure is one of the five elements of mastery as manipulatives were used to visualise the connections between the representations.

In regards of barriers for learning, the research instruments uncovered that manipulatives can be a distraction for individuals if the manipulatives are unfamiliar or not used for the correct mathematical purpose. This research finding supports the literature of Kelly (2006) as she described that manipulatives could be a distraction mechanism.

In conclusion, to overcome these barriers for learning the questionnaire table and semi-structured interview validated that manipulatives need to be carefully chosen for each mathematical concept. This finding justifies the literature of Furner and Worrell (2017) who acknowledged that Maria Montessori's approach to mathematics was by choosing carefully constructed materials to promote an individual's sense of mathematical understanding.

How do manipulatives impact the pupils' attitudes towards the mastery of mathematics in KS2?

The research instruments which were used to answer this research question is focus group interviews, with these responses being supported by questionnaires; semi-structured interviews and observations.

In relation to the pupils' attitudes, the responses from the focus group interview revealed that the pupils enjoyed mathematics more when using the manipulatives. This view is expanded upon in the questionnaires as out of the 8 teacher responses, 3 said they 'agreed,' that pupils enjoyed the mastery mathematics lessons when using manipulatives, while 4 said they 'strongly agreed,'.

In the lesson observation, pupils were engaged within the activity using the fraction blocks confidently to explain the multiplication of fractions back to the class. To further support this, in all four of the semi-structured interviews the teachers agreed that the manipulatives positively impacted the pupils' engagement. However, in the third semi-structured interview there was an acknowledgment of the teacher taking a facilitator role while pupils are engaged in the manipulatives.

The lesson observation highlighted that the pupils used the fraction blocks to address a misconception and this improved the pupil's self-esteem. This increase in pupil's self-esteem is stated in the second semi-structured interview as the participant explained that if pupils use the manipulatives as much as possible then they become comfortable and this consistency in using manipulatives improves the individuals' confidence and self-esteem.

With regards to the pupils' attitudes towards mastery mathematics, the responses from the focus group interview outlined that when using manipulatives, the pupils felt like they were creative and once the individuals completed a task then there was a sense of achievement.

Findings from the lesson observation and the semi-structured interviews have revealed that manipulatives can be used to increase pupils' self-esteem. This finding supports the literature from Carroll *et al* (2007) as when using manipulatives, individuals visualise abstract mathematical concepts which increases the individual's self-esteem.

The findings from the focus group interview identified manipulatives also promoted an individual's creativity and provided pupils with a sense of achievement. The promotion of an individual's creativity contrasts the literature of Carruthers and Worthington (2006) who established that there was a lack of creativity in mathematics. However, the sense of achievement when using manipulatives confirms the literature of Clements (2000) who stated that with clear instruction and exploration of manipulatives, the attitudes and achievement of individuals are improved in mathematics.

In conclusion, findings from the lesson observation and semi-structured interviews established that manipulatives provide pupils with confidence when engaged in the mastery lesson. This finding supports the literature of Brown and Liebling (2014) that manipulatives provide individuals with greater levels of resilience and confidence.

Brown and Liebling (2014) provide a useful insight by observing that individuals developed greater levels of resilience and confidence when using manipulatives, as it provided new and interesting representations of mathematical concepts. Furthermore, the third semi-structured interview distinguished that teachers must take a facilitator role when using manipulatives which supports the literature of Schoenfeld (2014) who clarifies that this facilitator role when using manipulatives enhances the individuals learning.

Conclusions & Recommendations

In conclusion, there is a complexity to the understanding behind mastery due to the four different identifications of the term: mastery approach; mastery curriculum; teaching for mastery and achieving mastery of areas in mathematics (NCETM, 2015). Respective of this, there is clear evidence through the findings and literature that manipulatives positively influence the teaching of mastery mathematics in primary schools. This research has impacted my teaching practice as a self-practitioner by allowing an insight into the key strategies of mastery mathematics, especially the use of manipulatives within the classroom. Furthermore, this research project has evidenced competence towards achieving Teachers' Standards 2d and 3b (DfE, 2013).

The recommendations from the research findings are:

The School

- Development for higher ability pupils to use manipulatives in mastery mathematics
- To ensure there is a reduction in distractions within the classroom setting when using manipulatives in mastery mathematics

Mastery Mathematics Programme

- Continual provision of mastery using a variety of representations (concrete to abstract)
- The programme needs to provide creativity, engaging and confidence building activities where the teacher acts in a facilitator role

The Government

- Deeper focus on the CPD for teachers adopting mastery mathematics at their schools
- To promote mastery mathematics through the acknowledgement of the potential progression of individuals when using mastery mathematics.

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